Managing Intellectual Capital and Innovation for Sustainable and Inclusive Society
27–29 May 2015
Bari • Italy
Management, Knowledge and Learning
Joint International Conference 2015
Technology, Innovation and Industrial Management

COMPARISON OF THE EFFICIENCY OF S-CONTROL CHART AND EWMA-S^2 CONTROL CHART FOR THE CHANGES IN A PROCESS

Supranee Lisawadi
Department of Mathematics and Statistics, Faculty of Science and Techonology, Thammasat University, Bangkok, Thailand
supranee@mathstat.sci.tu.ac.th

Punnamee Sachakamol
International Graduated Program for Industrial Engineering (IGP), Department of Industrial Engineering, Faculty of Engineering, Kasetsart University, Bangkok, Thailand

Abstract:
Statistical Process Control (SPC) is a method of quality control which uses statistical methods to monitoring and controlling the process to ensure that the manufacturing line operates at its full potential. At its full potential, the process can be applied to make as much conforming product as possible with minimum waste. However, the quality of the process control is determined by samples chosen and Control Chart tools whereas this research aims to determine and comparison the efficiency of S-Control Chart and EWMA-S^2 Chart. The determination factors are fluctuation in variances, different sizes of samples, and variations of sampling method. From 10,000 iteration run in R Version 2.0.2 with Median-Set Sampling, variances range from 0.0 to 3.0, the EWMA-S^2 Control Chart yield higher efficiency in every possible scenario tested.

Keywords: control chart, quality control, production, statistical process control, random sampling, standard deviation

1. THE ORIGIN AND SIGNIFICANCE OF THE PROBLEM

General manufacturing process tends to always vary. This variation will cause the quality of the product varies as well. The variation sometimes is not so much. Manufacturer allows this to happen because it does not affect the overall quality of the product. However, if the variation is severe and is not acceptable because it may make the product becomes a waste or cannot be used even though there have been attempts to reduce or control the variation in each factor, but it still has the variation eventually in the manufacturing process. Therefore, the variation should be controlled, which is the quality control to ensure that the quality of products meets the required standards or is in within the acceptable limit.

Statistical Process Control or SPC is the implementation of statistical process to control the manufacturing process so that the properties of products in the manufacturing process is consistent as well as raising manufacturing capacity or reducing the variation of the manufacturing process until the properties of products meet or close to the standards, which SPC now is very popular for analyzing the performance of the manufacturing process.

(Shewhart, 1939) has developed a control chart, which is considered as the main tool of SPC by relying on statistical methods. The main function of control chart is to detect variations in the manufacturing process and to be a tool to indicate how the manufacturing process operates, whether variation occurs out of control or is negligible, which is considered in control. This control chart, if the manufacturer uses to control the manufacturing process regularly, will prevent quality problems well since the control chart will detect changes in the manufacturing process in a timely manner allowing manufacturers to rapidly know manufacturing conditions. In the event of a problem, it can stop the manufacturing process and improve it.

In this study, 4 sampling plans will be used including Simple Random Sampling (SRS), Systematic Sampling (SYS), Ranked-Set Sampling (RSS) and Median-Set Sampling (MSS) by using R software package for simulating random sampling of each plan as a representative for monitoring and create a control chart.

From the study of the chart above, the researcher was interested to study the process with little change in the standard deviation with standard deviation control chart and exponentially weighted moving average control chart for examining the variation of the process financial. The study found that there was no comparison of the two control charts under the circumstances of the manufacturing process in the standard deviation with negligible change at the same time and the researcher will study different sample sizes in order to compare the performance of all control charts with the Average Run Length(ARL) at each sampling plan.

2. RELATED THEORIES AND RESEARCHES

In this study, the researcher was interested in several types of random sampling to create S control charts and exponentially weighted moving average control chart for measuring the variance of the process (EWMA-S^2 Control Chart) under data with a normal distribution by comparing control charts obtained from various sampling methods. The related theories and researches are as follows.

(Maravelakis & Castagliola, 2009) found that EWMA control chart is more effective than other charts if ARL1 is low for any constant in changes (τ) when ARL_0 are equal in all charts in which low λ value is suited to detect small to medium changes and high λ value is suited for large-scale change $(\tau \geq 1.8)$.

More researches are also compared the strength between the exponentially weighted Shewhart average control chart and synthetic by using Average Run Length (ARL) that is out of control as a basis on making a decision from the 2 conclusions. The distributions made the exponentially weighted average control chart the most effective.

(Tawantira & Mayureesawan, 2010) studied the control charts by Ranked-set Sampling and compared the ARL value from data simulation of 5 charts which were $\bar{x}-$chart, $\bar{x}-$RSS Chart, MRSS Chart, $\bar{x}-$MSS Chart, and MMSS Chart and showed the results at some m values and σ at all k levels and at specified δ value in the experiment plan. The results that the average of the manufacturing process has not changed (δ = 0) was found that at all levels of k, m, σ the MRSS-Chart had the best ability to monitor the manufacturing process since it gave the ARL value closest to the target value of 370 while $\bar{x}-$RSS, Chart, $\bar{x}-$MSS Chart, and MMSS Chart และ $\bar{x}-$Chart has subsequently lower ability to monitor the manufacturing process, having the ARL value more different from 370, respectively.

3. RESEARCH OPERATION

In this research the researcher wanted to study the performance of S-control charts and exponentially weighted moving average control chart for measuring the variance of a process (EWMA-S^2 Control Chart) for a process with a change in the standard deviation of various sampling methods by simulating data with different sampling methods. Then the average run length (ARL) was calculated using R software to compare the performance of both charts. The followings are the procedure to conduct the research.

3.1. Procedure

The procedure is divided as follows:

Data simulation:
Data simulation was performed using R software package version 3.0.2 to simulate data with normal distribution, average equals to 10, variance ranges from 0.0 to 3.0 and increases at 0.2 increment(0.0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0) for 200 samples (m = 200). Samplings were done using the following methods:

1. Simple Random Sampling (SRS)
2. Systematic Sampling (SYS)
3. Ranked-Set Sampling (RSS)
4. Median-Set Sampling (MSS)

The sample size was designated as 15, 20, 25, 30, respectively (n = 15, 20, 25, 30).

Specification of Control Range

1. Standard Deviation Control Chart (S Control Chart): The control range for S control chart was

$$LCL = B_3\bar{S}$$
$$UCL = B_4\bar{S}$$

in which B_3 and B_4 can be opened from the appendix of the Factors for Constructing Variables Control Charts.

2. Exponentially Weighted Moving Average Control Chart for Monitoring the Process Variance (EWMA-S^2 Control Chart): The control range for EWMA-S^2 Control Chart was

$$LCL = 0$$
$$UCL = \sigma_0^2 + c\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{\frac{2}{k}}\sigma_0^2$$

in which c is the critical value and k is the degree of freedom and k = n-1.
Average Run Length (ARL) is calculated from

$$ARL = \frac{1}{M}\sum_{i=1}^{M} RL_i$$

Where M is the number of experiment in each situation in order to find ARL,
RL_i is the number of samples monitored until the manufacturing process is out of control at experiment.

To find ARL, Monte Carlo simulation technique was used with the following procedure.

Standard Deviation Control Chart(S Control Chart)

1. Simulate data $X_{i,j} \sim N\left(10, \sigma^2\right)$ with variance from 0.0 to3.0 at 0.2 increment at n = 15, 20, 25, 30 for 200 samples.
2. Specify the lower control limit $LCL = B_3\overline{S}$ and upper control limit $UCL = B_4\overline{S}$
3. Calculate statistical values from the chart from

$$s_i = \sqrt{\frac{\sum_{j=1}^{n}\left(x_{i,j} - \overline{x}\right)^2}{n-1}}$$

or

$$s_i = \sqrt{\frac{n\sum_{j=1}^{n} x_{i,j}^2 - \left(\sum_{j=1}^{n}\left(x_{i,j}\right)^2\right)}{n(n-1)}}$$

4. Consider statistical value LCL < s_i < UCL (the statistical value is under control*)* when t = 1until LCL < s_i and s_i > UCL (the statistical value is out of control*)* and count the run length (RL).
5. Repeat steps 2, 3, and 4 until M = 10,000.
6. Calculate ARL.

Exponentially Weighted Moving Average Control Chart for Monitoring the Process Variance (EWMA-S² Control Chart)

1. Simulate data $X_{i,j} \sim N\left(10, \sigma^2\right)$with variance from 0.0 to3.0at 0.2 increment at n = 15, 20, 25, 30 for 200 samples.
2. Specify the upper control limit $UCL = \sigma_0^2 + c\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}\sqrt{\frac{2}{k}}\sigma_0^2$ where c = 3 and k = n-1
   Specify the lower control limit LCL = 0
3. Calculate Exponentially Weighted Moving Average Control Chart for Monitoring the Process Variance (EWMA) $z_i = (1-\lambda)z_{i-1} + \lambda v_i, i \geq 1$ where λ = 0.01 and $v_i = \ln s_i^2$.
4. Consider statistical value $LCL < z_i < UCL$ (the statistical value is under control) when t = 1 until $LCL < z_i$ and $z_i > UCL$ (the statistical value is out of control) and count the run length (RL).
5. Repeat steps 2, 3 and4until M = 10,000.
6. Calculate ARL.

3.2. Comparison of the chart efficiency:

In this study, ARL was used to compare the performance of S Control Chart and exponentially weighted moving average control chart (EWMA-S^2Control Chart) for measuring the process variance for a process that has a change in the standard deviation for various sample methods, where ARL is the average number of samples to be monitored until the discovery of the out of control and can be determined as follows:

- If ARL is high then the chart has low efficiency.
- If ARL is low then the chart has high efficiency.
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4. RESULTS

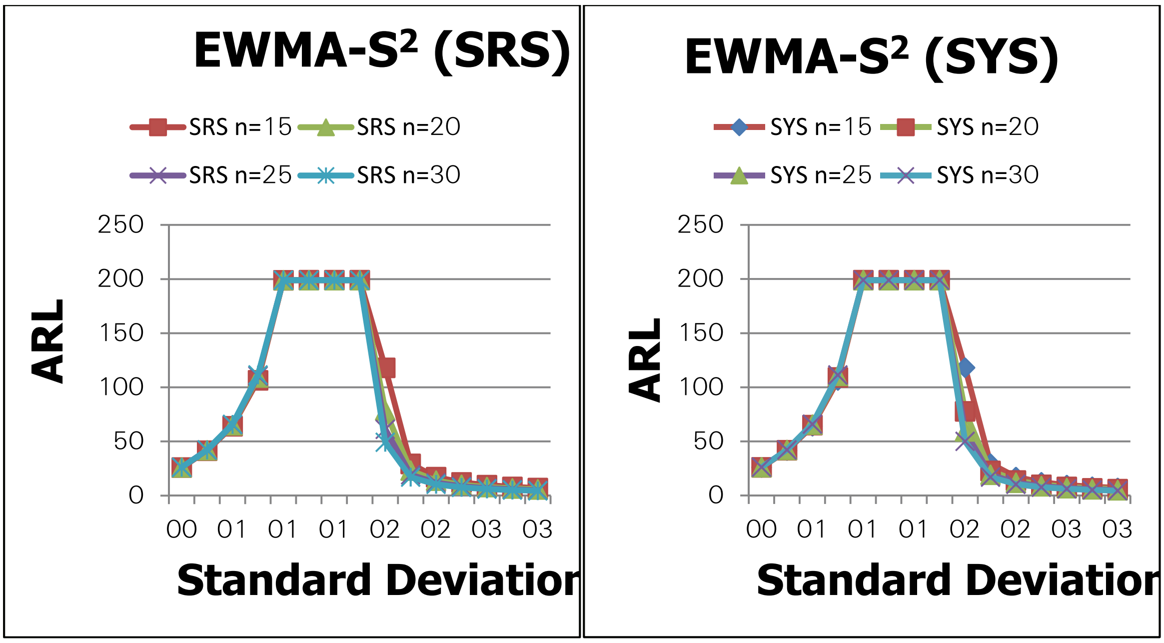
This research aims to study and compare the performance of S Control Chart and Exponentially Weighted Moving Average Control Chart (EWMA-S^2 Control Chart) for measuring the process variance in different situation such as when the data has a change in the standard deviation at different levels, the data has different sample sizes, and the data has different sampling methods in order to draw conclusions about the performance of S Control Charts and Exponentially Weighted Moving Average Control Chart (EWMA-S^2 Control Chart) for measuring the process variance using the average run length (ARL) as a value to compare the performance of both charts, which was presented in the form of tables and graphs showing the average run length (ARL).

From Tables 1-4, it was found that EWMA-S^2 Control Chart had the average run length (ARL) less than S Control Chart at all levelsof standard deviation and all sampling plans except RSS and MSS sampling methodindicating that EWMA-S^2 Control Chartis more efficientthan S Control Chart when the sample size is 30.

Table 1: ARL from control chart of different sampling sizes

	Sigma	SRS		SYS		RSS		MSS	
		S Control Chart	EWMA-S^2	S Control Chart	EWMA-S^2	S Control Chart	EWMA-S^2	S Control Chart	EWMA-S^2
n=15	1	151.17	198.91	154.05	198.96	0	22.30	153.60	17.92
	2	154.48	29.25	147.56	29.29	0	32.40	156.02	24.25
	3	152.94	6.93	151.87	6.93	0	40.22	152.42	30.57
n=20	1	151.34	199	149.13	198.97	0	22.34	156.04	17.98
	2	155.22	23.10	151.89	22.67	0	32.41	152.41	24.36
	3	154.56	5.76	152.66	5.78	0	40.41	153.77	30.62
n=25	1	157.14	199	154.26	199	0	22.48	148.69	17.99
	2	155.61	19.17	155.12	19.18	0	32.60	156.30	24.39
	3	149.77	4.97	151.52	4.90	0	40.46	153.16	30.72

Figure 1: ARL in the SRS and SYS



From figure 1, it was found that ARL in the Simple Random Sampling (SRS) and Systematic Sampling (SYS) will increase when the standard deviation rises and be constant when the standard deviation is between 1.0 to 1.6 and decrease when the standard deviation is greater than 1.6.

From Figure 2, Ranked-Set Sampling (RSS) and MSS give higher ARL when the standard deviation increases.

Figure 2: ARL in the RSS and MSS

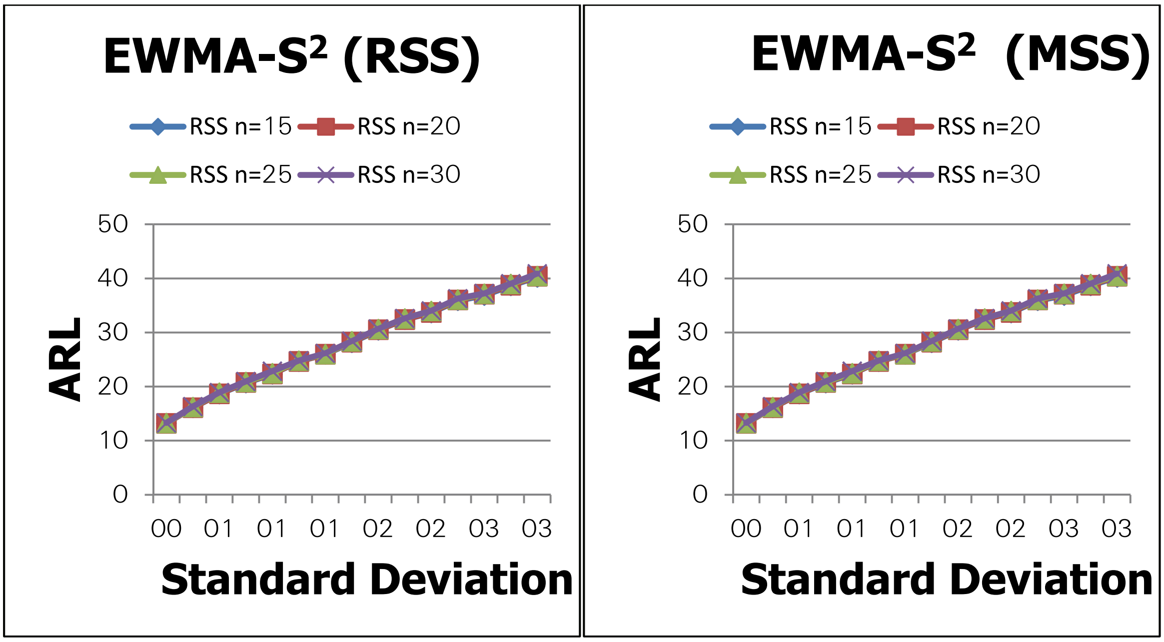
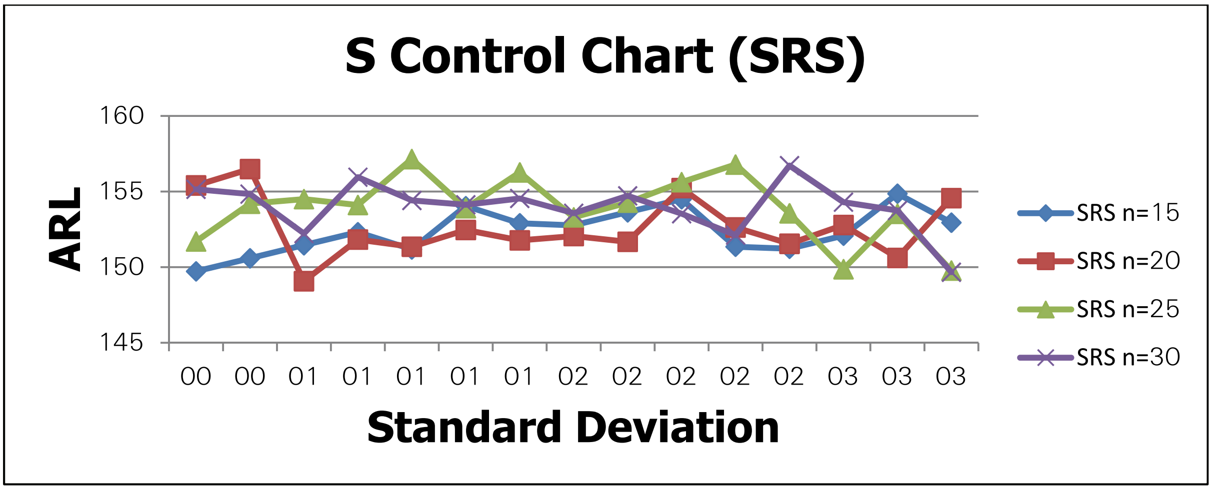
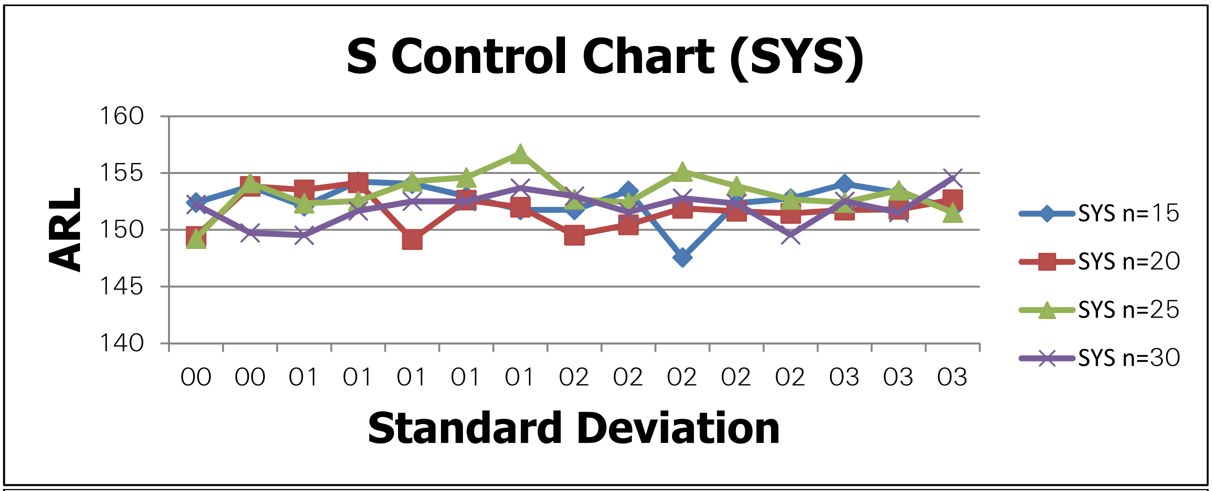
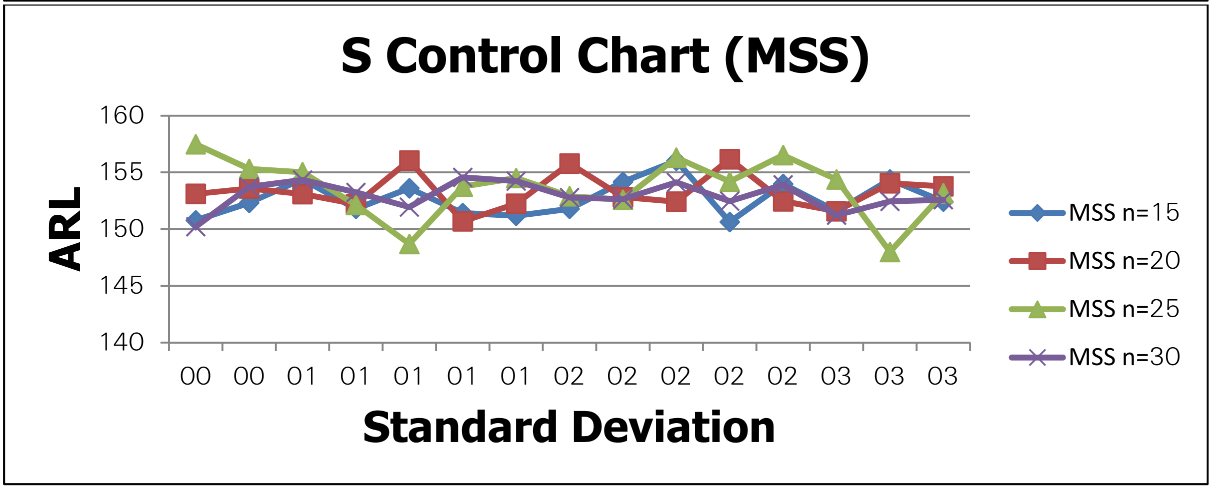


Figure 3: S Control Chart in the SRS, SYS, and MSS

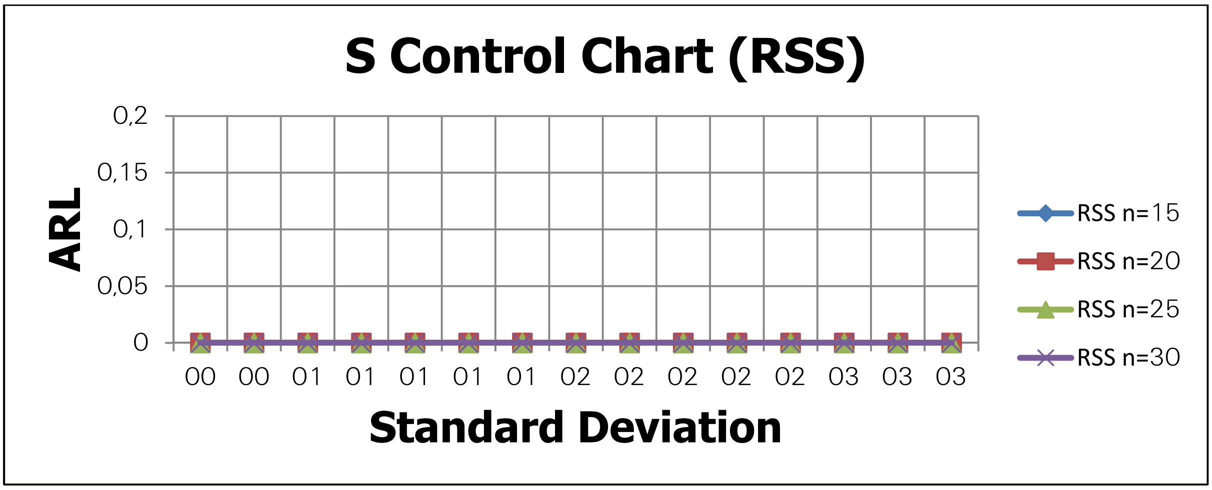






From figure 3, it was found that the sample size does not affect ARL of S-Chart, meaning the sample size does not affect the performance of S-Control Chart.

Figure 4: S Control Chart in the RSS



From figure 4, it was found that Ranked-Set Sampling (RSS) gives ARL = 0 at all sample sizes. This is because Ranked-Set Sampling (RSS) is a sampling method that sorts the value from low to high and uses the lowest number as the first sample and repeat until all samples are used. Therefore, sample number 1 is the sample that has the lowest observation. The lowest observation will be out of the control range when plotted on a graph resulting in ARL = 0.

The other figure are plots but not listed in this paper. The conclusions can be drawn as below:

1. It was found that SRS and SYS are more efficient than other sampling methods when the standard deviation is high. If the standard deviation is lower, MSS will be more efficient.
2. It was found that sampling method did not affect the performance of both charts because they gave the same ARL for all sampling methods.

5. CONCLUSIONS AND RECOMMENDATIONS

The findings in this research are to study and compare the performance of S Control Chart and Exponentially Weighted Moving Average Control Chart (EWMA-S^2 Control Chart) for measuring the process variance in different cases as follows. Data changed the standard deviation at different levels. Data had with different sample sizes. Sampling method was different for the data. The conclusions about the performance of S Control Chart and Exponentially Weighted Moving Average Control Chart (EWMA-S^2 Control Chart) for measuring the process variance using the average run length (ARL) as a value to compare the performance of two charts were that when considering the appropriate sampling method it was found that in the case of a minor variance ($\sigma^2 \leq 2.2$) Median-Set Sampling (MSS) was more efficient than other sampling methods, when applied to a large variance ($\sigma 2 > 2.2$) Simple Ransom Sampling (SRS) and Systematic Sampling (SYS) were more efficient than other sampling methods, and for when the sample size was larger, the control charts were also efficient control chart as well. In all cases, EWMA-S^2 Control Chart was more efficient than S Control Chart.

Table 2: The conclusion of the efficiency of the 2 control charts at various sample sizes.

Sample Size(n)	Most Efficient Control Chart
15	EWMA-S^2 Control Chart
20	EWMA-S^2 Control Chart
25	EWMA-S^2 Control Chart
30	EWMA-S^2 Control Chart

Table 3: The conclusion of the efficiency of the 2 control charts when the sampling method was different.

Sampling Method	Most Efficient Control Chart
SRS	EWMA-S^2 Control Chart
SYS	EWMA-S^2 Control Chart
RSS	EWMA-S^2 Control Chart
MSS	EWMA-S^2 Control Chart

In this study, normal distribution was used to simulate the data for the comparison of S Control Chart and Exponentially Weighted Moving Average Control Chart (EWMA-S^2 Control Chart) for measuring the process variance. If anyone is interested in conducting further study, other kinds of distributions may be used such as U-Shaped distribution or J-Shaped distribution, etc.

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