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# FIXED CHARGE UNBALANCED TRANSPORTATION PROBLEM IN INVENTORY POOLING WITH MULTIPLE RETAILERS

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**Abstract:**
The paper presents the formulation of a mathematical model of a multi-retailer inventory system with preventive lateral transshipment. The fixed charge unbalanced transportation problem is employed to find the optimal transshipment policy. Each retailer employs base stock periodic review policy. The model assumes Poisson distributed demand and instantaneous transshipment. Unsatisfied demands are fully backlogged. Transshipment quantity is controlled by the hold-back inventory level. Numerical experiments are conducted to illustrate the applicability of the proposed mathematical model. A comparative study between small and large pooling groups is also investigated. The results exhibit that the benefits of preventive lateral transshipment are more significant when the number of retailers increases. Moreover, by employing fixed charged transportation problem, the proposed model is proved to be effective in solving the problem with fixed transshipment cost for both identical and non-identical retailers.

*Keywords: inventory; inventory pooling; transshipment; preventive lateral transshipment; multiple retailers; fixed charge transportation problem*

# 1. INTRODUCTION

In a highly competitive market, manufacturers and retailers face randomness of customer demand. Stockout, which is caused by unpredicted demand, is considered critical in today's market. A win-win collaboration among members in supply chain is a strategy to better handle this issue. This strategy has been investigated by several studies such as Lehoux et al. (2010) to help effectively manage the inventory. However, the inventory problem becomes more crucial when the system has long replenishment lead time from a remote manufacturer which supplies inventory units to a number of retailers. The bullwhip effect, the phenomenon in which order variability increases when moving upstream, also make the inventory problem becomes more important in supply chain nowadays (Disney et al., 2007). In fact, all supply chains are now under pressure to increase or at least maintain their customer service level whilst reducing the operation cost.

Several approaches, such as emergency order and order expediting, have been proposed to evade such problems without resulting in excessive holding inventory. One approach in particular is to allow redistribution of inventory among retailers located in the same area. In this case, inventory is shipped from retailers with excess stock to retailers facing stockout. Such inventory movement is called lateral transshipment or inventory pooling which can be divided into two classes. First is emergency lateral transshipment which takes place after the realization of demand. It mainly aims to respond to the actual stockout. Second is preventive lateral transshipment which occurs at a specific point in the cycle before stockout is observed. The latter of inventory redistribution is intended to reduce stockout risk in the future. Lateral transshipment is justifiable when transshipment cost is lower than stockout cost and its lead time is shorter than the regular replenishment time. By employing lateral transshipment, performance improvement in terms of total system cost and customer service level can be achieved.

Motivated by this fact, this paper presents an extension of Yousuk and Luong (2013) by increasing the number of retailers in the system from two retailers to N retailers. The mathematical model is modified by including the fixed charge unbalanced transportation problem. The model is formulated using the expected path approach by considering all possible cases that are likely to occur in a replenishment cycle. All formulas are derived by conditioning on the on-hand inventory level of each retailer at transshipment point (i.e., larger or less than its hold-back inventory level). The key differences from other previous works are that the assumptions of identical retailers and negligible replenishment lead times are relaxed. Moreover, fixed transshipment cost is also included in the total system cost. Thus, the models presented in this paper can be applied in a more general inventory distribution problem.

In this paper, system allows inventory redistribution which is controlled by the hold-back inventory level at a specific time. It is obvious that the problem itself has a complex nature which results from the interrelations between lateral transshipment and inventory policy parameters. Due to this complexity and intractability, it is impractical to analytically obtain an optimal solution for the investigated system. Consequently, heuristic algorithm or simulation is often selected to determine the near optimal solution. Here, simulated annealing is employed to find solution for the preventive lateral transshipment problem.

# 2. BACKGROUND AND RELATED RESEARCHES

Lateral transshipment can be described as "the monitored movement of material between locations at the same echelon" (Özdemir et al., 2006). It is known to improve customer service level and to reduce the system cost. In the past decades, there exist many researches examining lateral transshipment problem or inventory pooling. Publications in this field have typically focused on the development of mathematical model and the solving methodology such as analytical approach, heuristic approximation, and simulation. Classification of these papers can be made along various aspects of the problem. Chiou (2008) conducted a survey on lateral transshipment researches and systematically categorized according to their assumptions, inventory and transshipment policy, performance measurements, and methodologies. Emergency lateral transshipment is the majority of publications in this field as cited in Archibald et al. (1997), Herer and Rashit (1999), and Olsson (2009). Since this paper mainly addresses a periodic base stock review policy with preventive lateral transshipment, only related studies with similar features will be discussed and summarized.

We begin with the literature review on preventive lateral transshipment. In this case, lateral transshipment takes place at the predetermined schedule before the entire demand is realized and served. The transshipment decision is based on equalization of inventory among retailers. At present, there exist only few publications on the subject. Gross (1963) allowed lateral transshipment at the beginning of replenishment cycle in a one-period, two-location inventory system. The optimal replenishment policy which depends on the beginning inventory level and cost structures was presented. The same distribution system was also studied by Karmarkar and Patel (1977). They expanded the work of Gross (1963) into multiple locations inventory system and took stocking decision into consideration. Their study confirmed that optimal solutions could be obtained only for small problems.

In order to maintain some level of realism in the model as it is unlikely that retailers would want to transship their entire inventory. Thus, the hold-back inventory level is introduced as threshold value for transshipment decision. This can be seen in Das (1975), Tagaras and Vlachos (2002), Xu et al. (2003), Zhao et al. (2006), Huang et al. (2007) and Yousuk and Luong (2013). Xu et al. (2003) developed a heuristic model that combines (Q, R) continuous review policy with the hold-back amount (H). Instead of focusing on minimizing total system cost as in our model setting, they focused on improvement of probability of no-stockout and demand fill rate. Zhao et al. (2006) investigated not only the transshipment decision but also the inventory decision. For tractability, it has been assumed that each individual retailer is modelled as a stocking point linked to a make-to-stock production facility which has exponential production rate and produces only after receiving a replenishment request. It is also assumed that the arrival of transshipment requests follow a Poisson process. In this research, the transshipment decision is not a one-time decision. Instead it should be considered at any point in time when demand of a single unit occurs. Huang et al. (2007) studied unilateral transshipment in a continuous review inventory system. They exhibited that such policy is suitable when retailers have highly different cost parameters. They determined the value of the hold-back inventory level by some specific rule in advance. However most of the literature examining the hold-back inventory level concept including the ones cited above assumed emergency lateral transshipment.

The papers that are most closely related to ours are Das (1975) and Tagaras and Vlachos (2002) and Yousuk and Luong (2013). For Das (1975), he studied the preventive lateral transshipment in two-retailer periodic inventory system. In his model, the replenishment lead time was ignored. He has found that by moving transshipment time to a fixed point in a cycle, the complete pooling is optimal. However, this claim may not correct in the system with positive replenishment lead time. This fact is the motivation for our paper. Thus, we consider the existence of positive replenishment lead time, and hence, the hold-back level is introduced to cover the demand during replenishment lead time. Our system is also similar to Tagaras and Vlachos (2002). However, the key different is that the hold-back inventory level is not an independent variable in their study. It is determined based on the average demand of the period from the transshipment point to the next regular order point plus a safety stock. As mention earlier, we treat the hold-back inventory level as a decision variable.

Yousuk and Luong (2013) presents mathematical model using of a two-retailer inventory system with preventive lateral transshipment. Each retailer employs base stock periodic review policy. The model assumes Poisson distributed demand and instantaneous transshipment. Unsatisfied demands are fully backlogged. Transshipment quantity is controlled by the hold-back inventory level. As mentioned earlier, we limit our attention to the expansion of this paper. Thus, this paper is extended into a more general platform of multi-retailer inventory system. In other words, it aims to address the formulation of a mathematical model representing the system with an infinite capacity warehouse and multiple retailers in which preventive lateral transshipments may occur in response to future stockout risk. Moreover, in this chapter, the presence of fixed lateral transshipment cost is considered in the proposed model.

In the multi-retailer system, there exists the possibility of multiple senders and receivers which results in the indecisive directions of transshipments. Thus, the appropriate transshipment policy should be carefully defined and investigated in order to precisely specify the transshipment directions and the transshipment quantity. In this paper, a strategy is proposed to address this difficulty by employing the fixed charge unbalanced transportation problem to help determine the optimal transshipment policy.

## 3. PREVENTIVE LATERAL TRANSSHIPMENT IN N-RETAILER INVENTORY SYSTEM

### 3.1. Model framework and assumptions

In Yousuk and Luong (2013), the research particularly focused on a two-retailer inventory system. This is limitation in real world application since most retail network involves many retailers. The proposed model for N-retailer inventory system in this paper is a modification of Yousuk and Luong (2013). In general, model framework and assumptions presented in Yousuk and Luong (2013) are also applied here. The key model framework and assumptions are summarized below.

Consider an inventory distribution system which consists of an infinite capacity warehouse and N retailers. The model assumes Poisson distributed demands. Moreover, unsatisfied demands are fully backlogged. All retailers operate under the base stock inventory policy with the same review cycle. The replenishment lead time from warehouse is nonnegligible, deterministic, and identical for all retailers.

Preventive lateral transshipment is instantaneous and occurs at a prespecified point. Transshipment quantity is influenced by the hold-back inventory level. The following assumptions are also used; 1) Transshipment will take place only before the replenishment order is placed. 2) Inventory sources are regular replenishment and lateral transshipment. 3) The probability that backorder occurs before the transshipment point is assumed to be very small and can be ignored.

The examined system behaves as follows:

1. At the beginning of the cycle, i.e., at point in time $L$ (regular replenishment lead time), the replenishment ordered in the previous period arrives at each retailer ($i$).
2. At the lateral transshipment point, each retailer reviews its remaining on-hand inventory level and decides whether to employ lateral transshipment. This decision is controlled by the on-hand inventory level ($OH_i$) and the hold-back inventory level ($R_i$). Lateral transshipments occur if there are some retailers having the on-hand inventory levels drop below their hold-back inventory levels while the other retailers having their remaining on-hand inventory levels above the hold-back inventory levels. The transshipment quantities are determined based on the differences between the remaining on-hand inventory levels and the hold-back inventory levels at all retailers. Therefore, there are three cases that need to be considered as follows:
    - *Case 1*: when $OH_i > R_i$ ; $i$ = 1, 2, …, $N$
      There will be no lateral transshipment since the remaining on-hand inventory levels are higher than the hold-back inventory levels at all retailers.
    - *Case 2*: when $OH_i \leq R_i$ ; $i$ = 1, 2, …, $N$
      There will be no lateral transshipment since the remaining on-hand inventory levels are lower than the hold-back inventory levels at all retailers.
    - *Case 3*: when $OH_i > R_i$ and $OH_j \leq R_j$ for some $i, j$; $i, j$ =1, 2, …, $N$, $i \neq j$
      There are lateral transshipments from some retailers $i$ to some retailers $j$ with the transshipment quantity ($X_{ij}$).
3. The demands occur after the lateral transshipment point until the next scheduled reorder point at each retailer are served.
4. At scheduled reorder point, each retailer places a regular order to bring its inventory level back to its order-up-to level ($S_i$).
5. The demands after the scheduled reorder point until the end of the cycle, i.e., the point in time $L+T$, at each retailer are then served. Unsatisfied demands during [$L+T-\tau$, $L+T$] are backlogged and will be served when the next replenishment arrives right after $L+T$.

**Figure 1:** Inventory level of sending retailer *i*

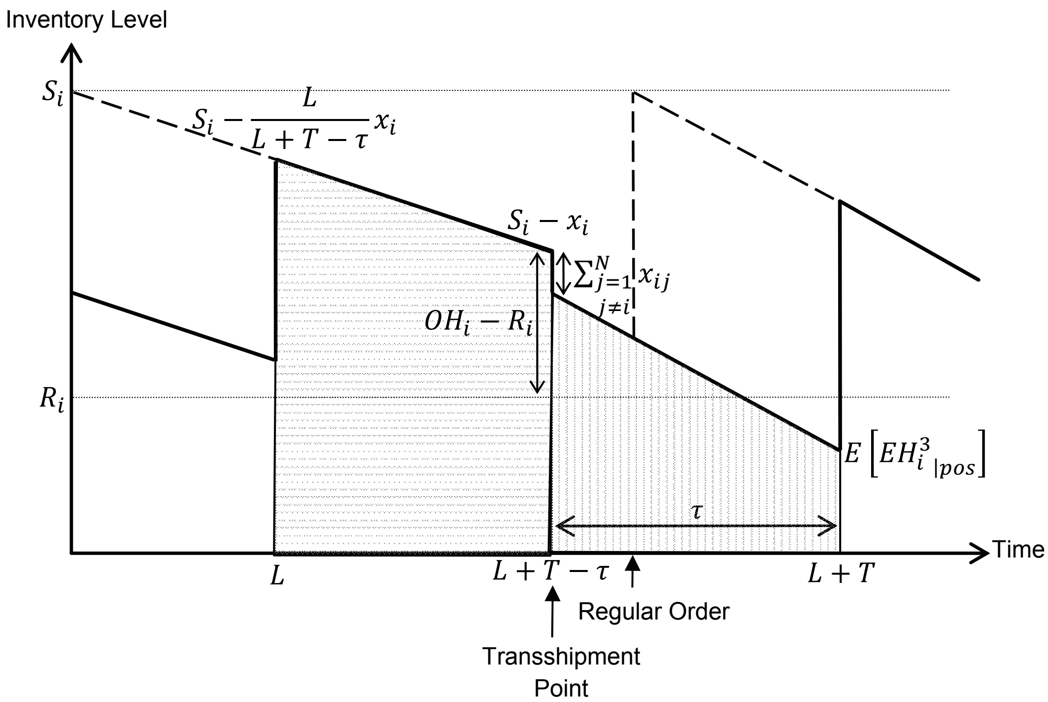
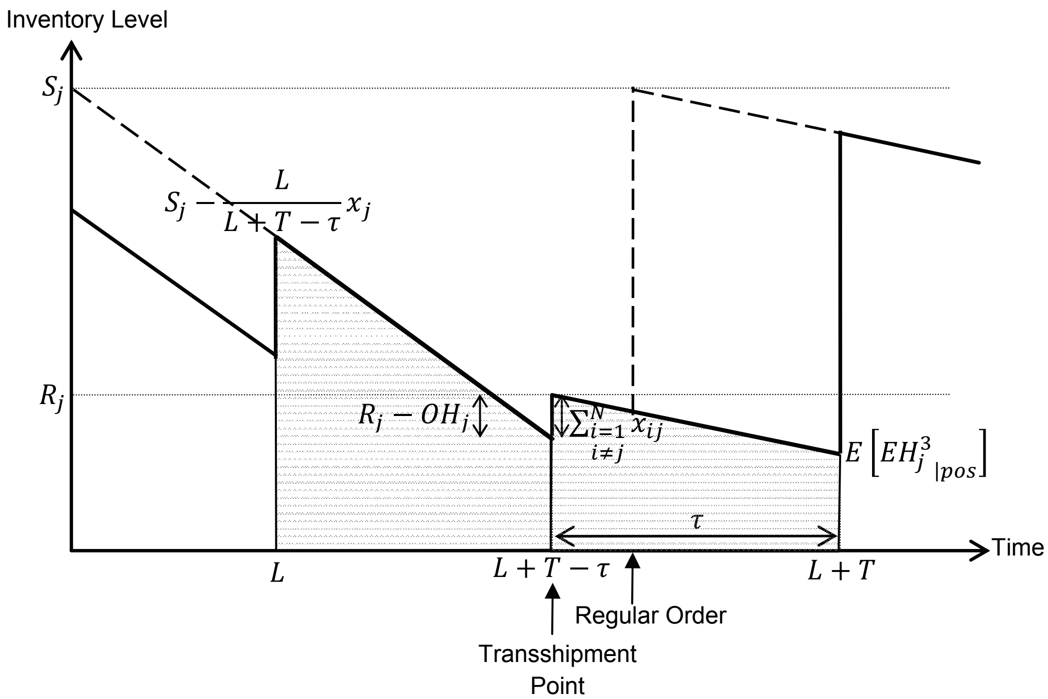



**Figure 2:** Inventory level of receiving retailer *j*




### 3.2. Derivation of the proposed model

This section presents the formulation of a mathematical model representing the system under consideration. Following the same derivations as in Yousuk and Luong (2013), the model is formulated using the expected path approach by considering all possible cases that are likely to occur in a replenishment cycle. For each case, the expected long-run total system cost is derived. However, since the expected path approach includes all possible scenarios that can occur in a cycle, the number of cases that needed to be studied are exponentially increased (e.g., four cases for two retailers, eight and sixteen cases for three and four retailers, respectively). Some approximation procedures should then be investigated to help simplify the mathematical model.

For case 1 and 2, there will be no lateral transshipment since all retailers have the remaining on-hand inventory levels higher or lower than their hold-back inventory levels. Therefore the model formulation is similar to Yousuk and Luong (2013).

In case 3, there are lateral transshipments from some retailers *i* to some retailers *j*. As shown in Figure 1 and 2, in order to calculated the expected total cost of case 3, the optimal transshipment quantity ($x_{ij}$) must be determined. In addition, in the multiple retailers system, the directions of transshipment become ambiguous, and hence, the appropriate sourcing rule which is the focus of this paper is also defined and investigated here. Due to space limitation, only the optimal transshipment quantity and transshipment direction are defined and presented in this paper. The fixed charge unbalanced transportation problem is employed to address this issue. *For more details of mathematical model formulation for all cases, please refer to Yousuk and Luong (2013)*.

A fixed charge unbalanced transportation problem is formulated and solved for the optimal transshipment quantity. It is noted that the total inventory transshipped from a specific retailer $i$ to retailers $j$ is $\sum_{\substack{j=1 \\ j\neq i}}^{N} x_{ij}$ and the total inventory transshipped to a specific retailer $j$ from retailers $i$ is $\sum_{\substack{i=1 \\ i\neq j}}^{N} x_{ji}$.

The additional notations for the fixed charge unbalanced transportation problem are presented below.

*Decision Variables for Transportation Problem:*

| | |
|---|---|
| $x_{ij}$ | Transshipment quantity from retailer $i$ to retailer $j$ |
| $y_i$ | Binary variable to decide whether retailer $i$ will send its inventory to other retailers |

*General Parameters and Variables:*

| | |
|---|---|
| $x_i$ | A specific value denoting demand at retailer $i$ during time interval $L+T-\tau$, i.e., from the time point when a regular order is placed to the next lateral transshipment point |
| $k_{ij}$ | Unit lateral transshipment cost from retailer $i$ to retailer $j$ |
| $K_i$ | Fixed lateral transshipment cost at retailer $i$ |

Given the base stock levels and the hold-back levels at retailers $i$, and a specific value of $x_i$ ($i = 1, \dots, N$), the fixed charge unbalanced transportation problem can be constructed in order to determine which routes to be opened and how many units to be transshipped on those routes. This transportation problem can be derived mathematically as follows.

The objective is to obtain the optimal transshipment quantities which minimize the total transshipment cost expressed in Equation 1.

$$TC^t = \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j\neq i}}^{N} x_{ij}\, k_{ij} + \sum_{i=1}^{N} K_i y_i \qquad (1)$$

Subjected to the following constraints (Equation 2-7):

- The inventory that can be shipped from sending retailers $i$ are limited by their hold-back levels. These constraints can be expressed as

$$\sum_{j=1}^{N} x_{ij} \quad \leq \quad S_i - R_i - x_i \qquad \forall i, i = 1, \dots, N, i \neq j \qquad (2)$$

- Receiving retailers $j$ can receive inventory in order to bring their on-hand inventory levels back to the hold-back levels. These constraints can be expressed as

$$\sum_{i=1}^{N} x_{ij} \quad \leq \quad R_j - (S_j - x_j) \qquad \forall j, j = 1, \dots, N, j \neq i, \qquad (3)$$

- Transshipment must always occur. This constraint is simply guarantee that transshipment quantities should be at least the minimum between available inventory from senders and demand from receivers. This constraint can be expressed as

$$\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N} x_{ij} \geq \; Min\left\{\sum_{\substack{i=1\\ i\neq j}}^{N}(S_i - R_i - x_i), \sum_{\substack{j=1\\ j\neq i}}^{N}(R_j - (S_j - x_j))\right\} \qquad (4)$$

- Nothing will be shipped from retailer $i$ if route from $i$ is not open. These constraints can be expressed as

$$\sum_{j=1}^{N} x_{ij} \leq My_i \qquad \forall i, i = 1, \dots, N, i \neq j \qquad (5)$$

- It is also required that all transshipments are nonnegative and $y_i$ is binary variable. These constraints can be expressed as

$$x_{ij} \geq 0 \qquad \forall i, j, \; i, j = 1, \dots, N, i \neq j \qquad (6)$$

$$y_i = 0, 1 \qquad \forall i, i = 1, \dots, N, i \neq j \qquad (7)$$

The above transportation problem will be solved to obtain the optimal transshipment cost, $TC^t$. This transshipment cost is then used to calculate the total system cost per cycle in case 3 corresponding to each pair of $x_i$ and $x_j$ ($i$, $j$ =1, 2, …, $N$; $i \neq j$). *For more details of mathematical model formulation for all cases, please refer to Yousuk and Luong (2013).*

## 4. RESULTS AND DISCUSSION

In this section, numerical experiments are conducted to investigate the benefits of preventive lateral transshipment in a larger retail network. The proposed mathematical model for a N-retailer inventory system is solved in order to obtain the optimal inventory and transshipment policy. Flow chart of the solution algorithm is presented in Figure 3.

Numerical examples are conducted to evaluate the impact of larger pooling group. It is obvious that in all cases, the expected long-run total system cost per cycle always decreases when lateral transshipment is employed. Moreover, the percentages of cost reduction are also compared in order to investigate the effect of a larger network and illustrated in Figure 4. The results lead to a conclusion that the benefits of preventive lateral transshipment are more significant when the number of retailers in a system increases. This can be explained as follows. When the number of retailers increases, the opportunity of inventory pooling also increases and there are more inventory for retailers to share to prevent future stockout risk.

By employing the fixed charge unbalance transportation problem to help find the appropriate transshipment policy, the proposed model can be efficiently applied to a system with fixed transshipment cost. As expected, the results show that the percentage of cost reduction decreases with higher fixed transshipment cost.

It has been concluded that the benefits of preventive lateral transshipment increase with the number of retailers in a system. However, this claim may not be true in nonidentical-retailer case. In order to further investigate, the numerical examples are extended to a nonidentical $N$-retailer inventory system. The conclusion can be drawn as follows;

1. Large pooling group continues to outperform small pooling group. However, it should be noted that all retailers have identical cost parameters. If pooling groups must be constructed in retail network with multiple members, retailers in the same pooling group should have the same average demand rate.
2. It might not be ideal to employ one large single pooling group when retailers in the system have different unit transshipment cost. In fact, based on the results in this example, retailers with low unit transshipment cost should be selected to form small single group if this is possible.
3. If one retailer has the lower unit transshipment cost in comparison to other retailers, it can be seen that this retailer will has high base stock level while the others have similar base stock value. Since unit transshipment cost from this retailer is lower, it will act as a central retailer and transship inventory to other retailers if needed.

**Figure 3:** Solution flow chart

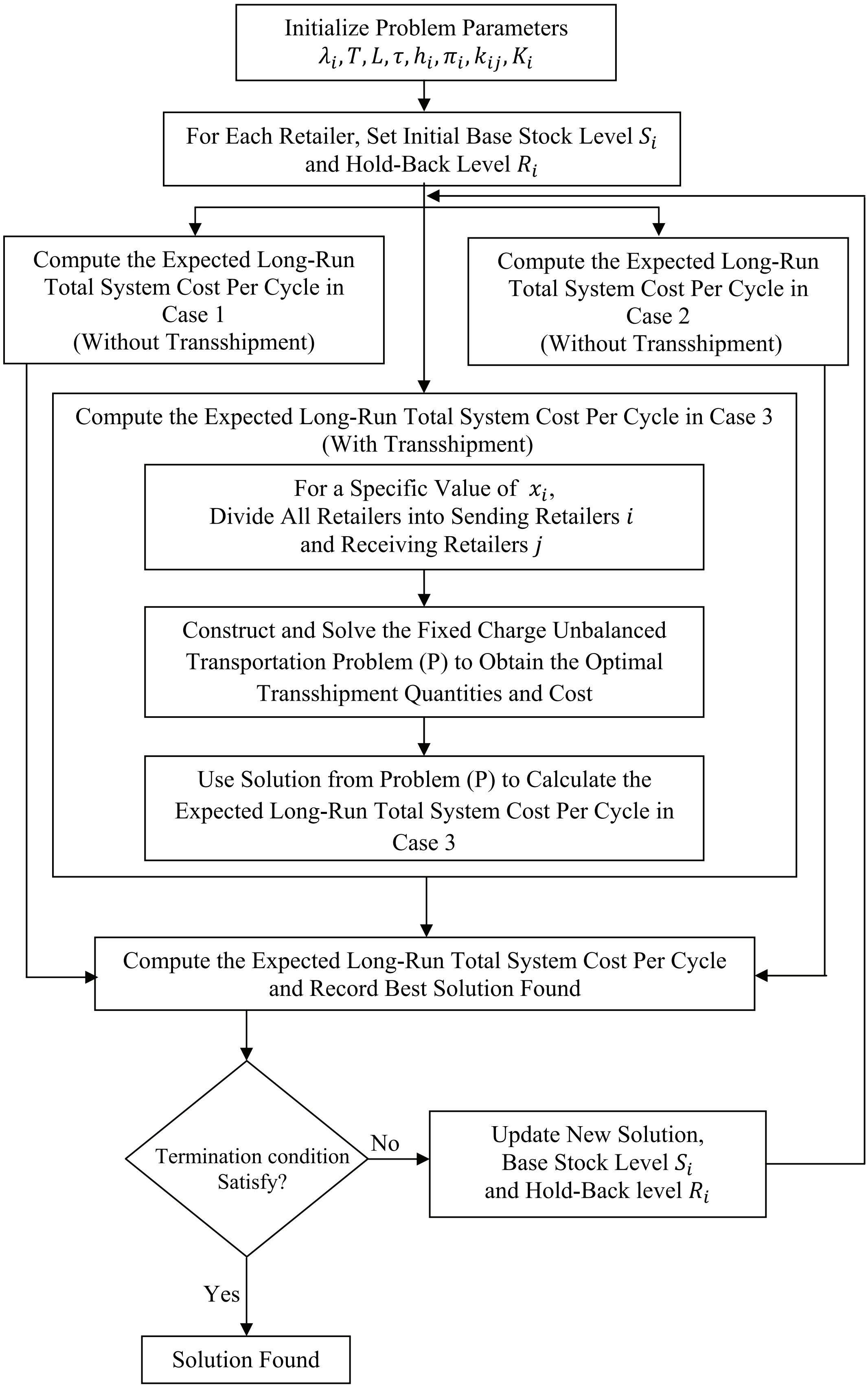
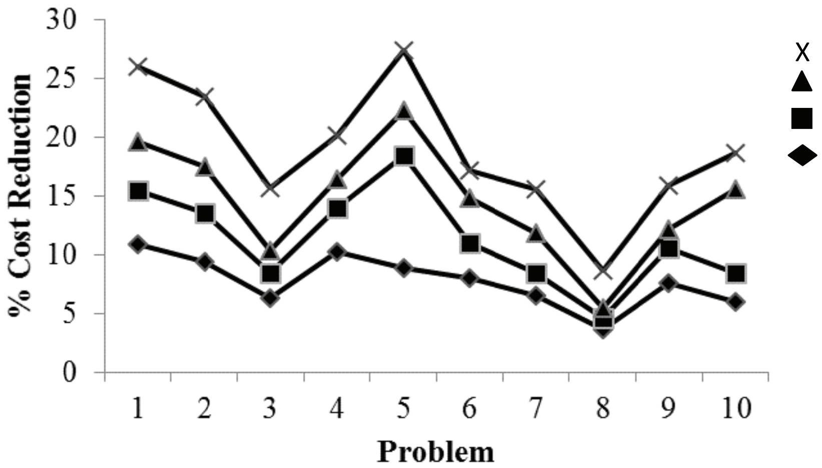


**Figure 4:** Comparison of the percentage of cost reduction with different number of retailers




## 5. CONCLUSION

This paper is an extension of Yousuk and Luong (2013). The N-retailer inventory network is investigated. The fixed charged transportation problem is employed to help determine the optimal transshipment policy. Numerical examples are conducted to evaluate the impact of larger pooling group. The results exhibit that in case of identical retailers, the benefits of preventive lateral transshipment are more significant when the number of retailers increases. The presence of fixed transshipment is also investigated as well as the nonidentical-retailer case. In the nonidentical-retailer case, three conclusions are drawn and illustrated in previous section.

Finally, it is worth mentioning that there are no definite rules in how to effectively construct transshipment pooling groups. It depends on the average demand rate and cost parameters. However, the mathematical models proposed in this paper are developed in a more general platform in which some assumptions in the previous works have been relaxed. Therefore, it is believed that the proposed models are more useful in practical application. However, solving each problem would be time consuming activity and also some of the assumption such as the predetermined cycle length and the transshipment time. These are considered the limitation of the study.

Therefore, for future research direction, both the cycle length and the transshipment time should be considered as the decision variables. But, it should be noted that including these variables into the model will make the model become too complicated. This is a drawback of the developed model presented here. However, if cycle length and transshipment time are considered as the parameters of the model, and then, sensitivity analysis is conducted to investigate the effects of these parameters, a good selection for these parameters can be achieved. In fact, related to the transshipment time, in most papers examining preventive lateral transshipment, the best transshipment point is still not fully investigated.

Furthermore, in order to decide whether the transshipment request should be preventive or emergency, some system characteristics such as the inventory policy and demand pattern should be taken into consideration. In addition, although the exact review period in periodic review inventory model is believed to provide a suitable opportunity for the prescheduled preventive lateral transshipment, but, a prescheduled transshipment may not be profitable in system with high demand variability. These issues should be incorporated in future research in this field.

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