A NOVEL PORTFOLIO SELECTION MODEL WITH PREEMPTIVE FUZZY GOAL PROGRAMMING

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Abstract:
In the Financial Markets, the investors mostly take into account the different kinds of the objectives to achieve the best performance. The multi-objective programming methods allow the investors to handle these objectives simultaneously. Preemptive fuzzy goal programming is one of the most efficient methods in the multi-objective programming, because it assigns the fuzzy goals to the objectives having different priorities among each other. In this article; the portfolio risk, the return levels and the beta coefficient defined in the Capital Asset Pricing Model are handled as different objectives. In order to assign the fuzzy goals to these objectives, the fuzzy membership functions are constituted with respect to the different types of investor strategies based on the market moving trends. By using these fuzzy membership functions in the preemptive fuzzy goal programming approach, a novel portfolio selection model is proposed. In the application sections, the two different periods having the upward and the downward moving trends in the Istanbul Stock Exchange National 30 Index are handled separately, then the optimal portfolios are determined using the proposed portfolio selection model in accordance with different investment strategies. Finally, the optimal portfolios are compared in terms of their performances based on the selling prices in the test periods.

Keywords: multiple objective programming, preemptive fuzzy goal programming, portfolio selection model, capital asset pricing model, portfolio management
1. INTRODUCTION

In finance theory, the principal phenomenon is risk. For this reason, the evaluation of risky assets is one of the major research tasks in finance for years. Although well-known classic portfolio selection models such as mean-variance model of Markowitz (1952) and mean-absolute deviation model of Konno and Yamazaki (1991) minimize risks defined in their objective function at any given level of the expected return, there are some significant factors such as the liquidity, the currency exchange risk, and the movement of financial market that have substantial effects on the investment decision exception of risk-return tradeoff. If the liquidity and the currency exchange risk are overlooked in the investment process, the movement of financial market can be taken into account with beta coefficient according to investment strategies of investors. For this reason, beta coefficient should be used as a goal or a restriction in any portfolio selection model (Kocadağlı and Cinemre, 2010).

In the investment decisions; risk, return and beta coefficient of any portfolio can be considered as separate objectives. As known from goal programming, the decision makers might desire to assign the different priorities to their objectives (Chen and Tsai, 2001; Wang and Fu, 1997, Hu et al., 2007). The decision makers are able to assign the fuzzy goals with the different priorities to these objectives. Thus, the portfolio selection problem can be transformed into fuzzy goal programming. In the real life problems, the fuzzy goal programming plays an important role because of having flexibility on all coefficients and resources unlike deterministic approaches.


The purpose of this study is to develop a novel portfolio selection model that includes the multiple objectives having certain priorities among each other. For this reason; risk, return and beta coefficient are determined as the objectives of portfolio selection model having the fuzzy goals. By using the membership functions of risk, return and beta coefficient in the preemptive fuzzy goal programming model introduced by Chen and Tsai (2001) and considering different types of investor strategies, a novel portfolio selection model is developed. In order to examine the implementations of this novel portfolio selection model, upward (bullish) and downward (bearish) moving trends in Istanbul Stock Exchange National 30 Index (ISE30) are handled separately.

2. FUZZY GOAL PROGRAMMING MODEL

In order to achieve better investment performance, the decision makers inherently assign the preemptive priorities to some objectives that have higher priorities than the others under system constraints. According to Chen and Tsai (2001), the fuzzy goals are ranked into the desired priority levels as follow:

\[
\text{Priority Level 1: } \{G_{1i}\} \mid i = [1,2,...,m] \\
\text{Priority Level 2: } \{G_{2i}\} \mid i = [1,2,...,m] \\
\cdots \\
\text{Priority Level m: } \{G_{mi}\} \mid i = [1,2,...,m] \\
\]

(1)
where \( G_{m,2} \), \( G_{m,2} \) and \( G_{m,2} \) are the disjoint sets of fuzz goals \((m \leq p)\). According to above preemptive priority structure, the relationship among the fuzzy goals can be arranged as follows:

\[
P_{1,2} \geq P_{2,1} \\
\ldots \\
P_{m,2} \geq P_{m,1}
\]

(2)

where \( P_{1,2} \) is the linear membership function that corresponds to \( i^{th} \) fuzzy goal in the \( i^{th} \) priority level. In order to find a set of solutions that satisfies above desired structure in Eq. (2) under the system constraints \( Ax \geq C \), the sum of each fuzzy goal’s achievement degrees can be maximized as follows (Chen and Tsai, 2001):

**Model 1**

\[
\text{Max } \sum_{i=1}^{m} \sum_{j=1}^{p} \mu_{ij} \\
\text{s.t.} \\
P_{1,2} \geq \mu_{2,1} \\
\ldots \\
P_{m,2} \geq \mu_{m,1} \\
Ax \geq C \\
\mathcal{I} \subseteq \{1, 2, ..., p\}
\]

(3)

Here, \( P_{1,2} \)’s are functions by using Model 1, the decision makers are able to find an optimum solution accordance with their preemptive priority structure among the fuzzy goals. In the decision theory, the characters of decision makers can be divided into three categories of risk-averse, risk-seeking and risk-neutral. If decision makers achieve their goals at least (at most) to a certain level, a unit less than (more than) that level will cause a lower degree of satisfaction for a risk-seeker than that for a risk-averse (Wang and Fu, 1997). In the general fuzzy goal programming structure, the fuzzy constraints can be defined as follows:

\[
G_{ij} : g_{ij}(x_1, x_2, ..., x_n) \equiv (Ax)_{ij} = B_{ij} \quad \text{(around)}
\]

(4)

\[
G_{ij} : g_{ij}(x_1, x_2, ..., x_n) \equiv (Ax)_{ij} \leq B_{ij} \quad \text{(at most)}
\]

(5)

\[
G_{ij} : g_{ij}(x_1, x_2, ..., x_n) \equiv (Ax)_{ij} \geq B_{ij} \quad \text{(at least)}
\]

(6)

\( x \geq 0; i = 1, 2, ..., m; j = 1, 2, ..., p \)

Here, the symbols “\( \equiv \)”, “\( \leq \)” and “\( \geq \)” specify the fuzzified aspiration levels with respect to the linguistic terms of “around”, “at most” and “at least” defined in Eq. (1), (2) and (3) respectively (Wang and Fu, 1997). If desired, the different kinds of membership functions can be used in accordance with the strategies of decision makers. For instances; if \( c_{ij} = 1 \), then \( \left( \mu_{ij}(Ax) \right)^{c_{ij}} \) is a piecewise linear membership function. Otherwise, it is a nonlinear membership function with \( c_{ij} = 2 \) of contraction or \( c_{ij} = 0.5 \) of dilation. As a result of contraction and dilation, the fuzzy goal programming transforms into either the linear or nonlinear programming. According to properties of dilation, contraction and motionless of membership functions, Model 2 can be solved by different characters of decision makers that correspond to risk-averse, risk-seeking, and risk-neutral respectively. Let’s use the exponents of the membership functions for the decision maker’s strategies, Model 1 can be arranged as follows:

**Model 2**

\[
\text{Max } \sum_{i=1}^{m} \sum_{j=1}^{p} \mu_{ij} \\
\text{s.t.} \\
P_{1,2} \geq \mu_{2,1} \\
\ldots \\
P_{m,2} \geq \mu_{m,1} \\
Ax \geq C \\
\mathcal{I} \subseteq \{1, 2, ..., p\}
\]

(7)
By means of Model 2, the decision makers not only use the preemptive priority for the fuzzy goals, but also solve this model with respect to the different kinds of strategies. Besides, it is possible to use the three types of constraints in Model 2 based on Eq. (4), Eq. (5) and Eq. (6). That is, the decision makers can construct different kinds of membership functions depending on the constraint types, and then solve the problem accordance with their strategies.

3. CONSTRUCTING MEMBERSHIP FUNCTION

3.1. Membership Function of Risk

According to behavior of risk aversion, the membership function of portfolio risk, it can be constructed as a monotonically decreasing piecewise-linear function to the risk levels (Kocadağlı, 2006). Let \( Z_0 \) and \( Z_1 \) be the bounds of portfolio risk that correspond to minimum and maximum objective function values of Markowitz’s (1952) model respectively. Here, \( Z_0 \) and \( Z_1 \) can be solved directly from Markowitz’s (1952) model at the minimum and the maximum return levels respectively. Thus, the membership function of portfolio risk and its graph can be given as follows (Kocadağlı, 2006), (Keskin, 2013):

Figure 1: Membership Function of Risk

\[
\mu(x) = \begin{cases} 
1 & , \ Z < Z_0 \\
1 - \frac{\sum_{i=1}^{n} x_i \sigma_{ij} - Z_0}{Z_1 - Z_0} & , \ Z_0 \leq Z \leq Z_1 \\
0 & , \ Z > Z_1 
\end{cases}
\]

(8)

3.2. Membership Function of Return

Let’s consider that the return rate desired over an asset or portfolio begins to take the larger values than expected one. In these cases, having larger satisfaction levels of investors is quite plausible since they behave more willing to return. For this reason, the membership function of return can be constituted as a monotonically increasing piecewise-linear function to return levels. Let \( R_{\text{min}} \) and \( R_{\text{max}} \) be minimum (or expected) and maximum return respectively. Thus, the membership function of return and its graph can be given as follows (Kocadağlı, 2006), (Keskin, 2013):

Figure 2: Membership Function of Return

\[
\mu_R(x) = \begin{cases} 
0 & , \ \sum_{i=1}^{n} x_i < E(R_{\text{min}}) \\
\frac{E(R_{\text{max}}) - \sum_{i=1}^{n} x_i}{E(R_{\text{max}}) - E(R_{\text{min}})} & , \ E(R_{\text{min}}) \leq \sum_{i=1}^{n} x_i \leq E(R_{\text{max}}) \\
1 & , \ \sum_{i=1}^{n} x_i > E(R_{\text{max}}) 
\end{cases}
\]

(9)

3.3. Membership Function of Beta Coefficient

Beta is generally estimated by using the standard market model which is defined as the following linear regression model (Kocadağlı, 2013):
\[ R_t = \alpha_i + \beta_i R_{mt} + \varepsilon \]  \hspace{1cm} (10)

where \( R_t \) is the realized return on stock \( i \) over interval \( t \); \( R_{mt} \) is the realized return on the market index over interval \( t \); \( \alpha_i \) is the constant term for stock \( i \); \( \beta_i \) is the sensitivity of stock \( i \) returns to the market index returns measured as the covariance between the stock return and the market portfolio return \( \frac{\text{cov}(R_i, R_m)}{\text{var}(R_m)} \). According to Eq. (10), the market return \( R_{mt} \) and the stock return \( R_t \) can be defined as independent and dependent variables respectively.

There are some useful interpretations with respect to beta values. For instance; if \( \beta = 1 \), it is assumed that the movement of stock is generally in the same direction as, and about same amount as the movement of financial market. A beta of greater than 1 indicates that the movement of stock is generally in the same direction as, but more than the movement of market. For \( 0 < \beta < 1 \), the movement of stock is generally in the same direction as, but less than the movement of market. The negative values of beta can be interpreted as that a stock having negative beta generally moves in the opposite direction according to the market index. In the other hand, it is possible that a stock having the negative beta might bring profit even if the market shows downward moving trend (Kocadağlı and Cinemre, 2010), (Kaya and Kocadağlı, 2012).

From the explanations mentioned above, it can be concluded that the investors usually prefer to the stocks or the portfolios having beta values greater than 1 if predicted that the market would show the upward moving trend in the investment period. Therefore, the membership function corresponding to the upward moving trend can be constructed as a monotonically increasing piecewise-linear function, and its graph can be drawn as follows (Kocadağlı and Cinemre, 2010), Keskin (2013):

**Figure 3:** Membership function for upward moving trend

\[
\mu_{\beta}(x) = \begin{cases} 
0 & \text{, } \sum_{i=1}^{n} \beta_i x_i < \beta_{\min} \\
1 - \frac{\beta_{\max} - \sum_{i=1}^{n} \beta_i x_i}{\beta_{\max} - \beta_{\min}} & \text{, } \beta_{\min} \leq \sum_{i=1}^{n} \beta_i x_i \leq \beta_{\max} \\
1 & \text{, } \sum_{i=1}^{n} \beta_i x_i > \beta_{\max}
\end{cases} \hspace{1cm} (11)
\]

In the downward moving trend case, the investors should prefer to the stocks or portfolios having negative beta values. Therefore, the membership function can be constituted as a monotonically decreasing piecewise-linear function, and its graph can be drawn as follows (Kocadağlı and Cinemre, 2010), Keskin (2013):

**Figure 4:** Membership function for downward moving trend

\[
\mu_{\beta}(x) = \begin{cases} 
1 & \text{, } \sum_{i=1}^{n} \beta_i x_i < \beta_{\min} \\
1 - \frac{\sum_{i=1}^{n} \beta_i x_i - \beta_{\min}}{\beta_{\max} - \beta_{\min}} & \text{, } \beta_{\min} \leq \sum_{i=1}^{n} \beta_i x_i \leq \beta_{\max} \\
0 & \text{, } \sum_{i=1}^{n} \beta_i x_i > \beta_{\max}
\end{cases} \hspace{1cm} (12)
\]

**4. CONSTRUCTING PORTFOLIO SELECTION MODEL**

Let the membership functions of risk, return and beta put into the Model 2 proposed in Eq. (7), the following portfolio selection model can be constituted:
Model 3:
Max \( Q = \mu_{\alpha} + \mu_{R} + \mu_{B} \)
s.t.
\[
\begin{align*}
0.5 \leq \mu_{\alpha} & \leq 2, \\
0 \leq \mu_{R} & \leq 1, \\
0 \leq \mu_{B} & \leq 1,
\end{align*}
\]
\[
\sum_{i=1}^{n} x_{i} = M_{0}, \quad 0 \leq x_{i} \leq M_{0} \quad (i = 1, 2, ..., n).
\]

where \( Z, R \) and \( B \) are the abbreviations of Risk, Return and Beta respectively. Here, the decision makers desire the larger the objective function values to increase the satisfaction levels with respect to the preemptive fuzzy goals. Besides, if desired, this model can be solved for different kinds of strategies corresponding to risk-averse, risk-seeking and risk-neutral using the properties of dilation \((c = 0.5)\), contraction \((c = 2)\) and motionless \((c = 1)\) of membership functions respectively.

5. APPLICATIONS

In the implementations, to analyze how investors should behave in accordance with market moving trends, two terms in the ISE30 index are handled separately. In the first implementation, an interval having downward trend in the ISE30 is preferred. In the second implementation, to analyse a different trend case, an interval having the upward trend is used. The sample data belonging these terms includes the closed prices in the opening and closing sessions of stocks traded in ISE30 between January 2011 and March 2011. In the analyses, the statistics required in the portfolio selection processes and the beta coefficients of stocks were evaluated using Matlab2012 package program. The portfolio selection problems were solved using the Solver Toolbox in the Excel 2010. In both implementations, the total fund \( M_{0} \) is taken as 100 Turkish Liras (₺). According to Stocks traded in ISE30, the return levels realized in January and March are given in Table 1. The min and max values of beta coefficient realized in the two periods are given in the Table 2. The priority levels to risk classes are showed in the Table 3.

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>RETURN LEVELS IN PERIODS (%)</th>
<th>TABLE 2</th>
<th>BETA’S IN THE PERIODS</th>
<th>TABLE 3</th>
<th>THE PRIORITY LEVELS TO RISK CLASSES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Return levels (%)</strong></td>
<td><strong>Beta Coefficients</strong></td>
<td><strong>Periods</strong></td>
<td><strong>Min</strong></td>
<td><strong>Ave</strong></td>
<td><strong>Max</strong></td>
</tr>
<tr>
<td>Jan.</td>
<td>-0.94</td>
<td>-0.02</td>
<td>1.13</td>
<td>Z ≥ R ≥ β</td>
<td>Z ≥ R ≥ β</td>
</tr>
<tr>
<td>March</td>
<td>-0.85</td>
<td>0.30</td>
<td>1.15</td>
<td>Z ≥ R ≥ β</td>
<td>Z ≥ R ≥ β</td>
</tr>
</tbody>
</table>

5.1. Analysis of downward moving trend in ISE30 index

In order to analyze the investor behaviors when market has a downward moving trend, the sample in January 2011 of ISE30 index illustrated in Fig.5 is handled. The sample in February 2011 illustrated in Figure 6 is left as a test period to measure the performances of selected portfolios. Thus, the optimum portfolios evaluated with the proposed model and Markowitz’s model are compared in terms of their returns over the test period. The first step of constructing the portfolio selection model is to determine the most appropriate membership functions with respect to market trend and investor’s strategy. Therefore, Markowitz’s model was solved for min and max return rates realized in January 2011 separately, and then \( Z_{0} \) and \( Z_{1} \) were evaluated as 0.68 and 29.16 respectively. Thus, the membership function of risk was constituted by using bounds \( Z_{0} \) and \( Z_{1} \) in Eq. (8). To set the membership function of return in Eq. (9); min and max return rates in Table 1 were used. Because of downward moving trend of ISE30, the membership function of beta was constituted as a monotonically decreasing
Table 4: Selling prices of portfolios (€) in test period

<table>
<thead>
<tr>
<th>Risk Levels</th>
<th>Proposed Models</th>
<th>Risk Aversion</th>
<th>Risk Seeker</th>
<th>Markowitz</th>
</tr>
</thead>
<tbody>
<tr>
<td>02</td>
<td>100.7</td>
<td>101.5</td>
<td>101.7</td>
<td>102.4</td>
</tr>
<tr>
<td>04</td>
<td>100.7</td>
<td>100.8</td>
<td>101.2</td>
<td>102.4</td>
</tr>
<tr>
<td>07</td>
<td>100.6</td>
<td>101.0</td>
<td>101.6</td>
<td>103.6</td>
</tr>
<tr>
<td>08</td>
<td>100.8</td>
<td>101.3</td>
<td>101.8</td>
<td>103.6</td>
</tr>
<tr>
<td>09</td>
<td>101.0</td>
<td>101.7</td>
<td>101.9</td>
<td>102.4</td>
</tr>
<tr>
<td>10</td>
<td>100.4</td>
<td>100.6</td>
<td>100.8</td>
<td>101.2</td>
</tr>
<tr>
<td>14</td>
<td>100.6</td>
<td>100.5</td>
<td>100.4</td>
<td>100.0</td>
</tr>
<tr>
<td>15</td>
<td>101.4</td>
<td>101.6</td>
<td>101.5</td>
<td>101.2</td>
</tr>
<tr>
<td>16</td>
<td>101.0</td>
<td>101.3</td>
<td>101.3</td>
<td>101.2</td>
</tr>
<tr>
<td>17</td>
<td>100.8</td>
<td>100.4</td>
<td>100.5</td>
<td>100.0</td>
</tr>
<tr>
<td>18</td>
<td>102.4</td>
<td>101.9</td>
<td>100.0</td>
<td>101.5</td>
</tr>
<tr>
<td>22</td>
<td>98.93</td>
<td>99.00</td>
<td>98.99</td>
<td>98.93</td>
</tr>
<tr>
<td>23</td>
<td>96.97</td>
<td>98.12</td>
<td>97.05</td>
<td>97.27</td>
</tr>
<tr>
<td>24</td>
<td>90.33</td>
<td>90.63</td>
<td>90.35</td>
<td>89.16</td>
</tr>
<tr>
<td>25</td>
<td>91.15</td>
<td>90.99</td>
<td>91.41</td>
<td>92.77</td>
</tr>
<tr>
<td>28</td>
<td>91.48</td>
<td>90.75</td>
<td>91.22</td>
<td>92.77</td>
</tr>
<tr>
<td>E(R)</td>
<td>98.97</td>
<td>99.43</td>
<td>99.37</td>
<td>98.92</td>
</tr>
</tbody>
</table>

5.2. Analysis of upward moving trend in ISE30 index
In the second implementation, the performance in the upward moving trend case of the proposed approach was investigated. In analysis, the sample in March 2011 illustrated in Fig. 7 was handled. To compare the performance of portfolios, the sample in April 2011 illustrated in Figure 8 was used as test period. After Markowitz’s model was solved for min and max returns realized in March 2012 separately, $Z_0$ and $Z_1$ were evaluated as 0.52 and 19.51. Thus, the membership function of risk is constituted by using $Z_0$ and $Z_1$ values. To set the membership function of return in Eq. (9); min and max return values in Table 1 were used. Because of upward moving trend of ISE30, the membership function of beta was constituted as a monotonically increasing piecewise-linear function given in Eq. (12). The proposed portfolio selection model was solved with respect to priorities and risk classes given in Table 2. In order to compare the performances of the portfolio selection models, Markowitz’s model was solved for expected return level (average return) and maximum of average returns of all stocks in March 2011. Lastly, the performances of portfolios were analyzed in terms of average returns realized in April (test period), and then these performances were given in Table 5.

![Figure 7: ISE30 index in March 2011](image7)

![Figure 8: ISE30 index in April 2011](image8)
6. RESULTS AND DISCUSSION

In the first implementation, the different priorities are considered for the fuzzy goals, and then the proposed portfolio selection models are solved in accordance with three types of investor behaviors for downward moving trend in ISE30. The portfolio performances based on the selling prices realized daily in the test period are given in Table 4. From these results, it can be concluded that if the investors use the proposed models with priority, they would bring profit, even if ISE30 has downward moving trend. Especially, it is possible to get the positive returns until February 18 (for the first 13 days), even if average return realized over test period (totally 19 days) is negative (\(E(R) < M_0=100\)). Besides, if the investors put the beta goal into first priority, they gain much more returns than other priority strategies as well. However, Markowitz’s model fails to determine the efficient portfolios because it is not able to take into account the market trends.

In the second implementation, the different priorities are considered for the fuzzy goals as similar to first implementation, and then the proposed portfolio selection models are solved in accordance with three types of investor behaviors for upward moving trend in ISE30. The portfolio performances of risk classes based on the selling prices realized over test period (April) are given in Table 5. From these results, it can be concluded that the risk-seeker investors would bring profit much more than the others. Besides, if the investors put the beta goal into first priority, they gain much more returns than other priority strategies as well. Here, Markowitz’s model carries out fewer returns because it is based on conservative strategy even if ISE30 index has upward moving trend. Although Markowitz’s model...
was solved at the greater return levels to get much more profit, its portfolio gave less return than the others.

7. CONCLUSION

According to the analysis results, considering the market movements in the portfolio selection problems assures more realistic approach. Especially, beta coefficient allows the decision makers to take into accounts the market movements. For this reason; the beta goals using together with couple of risk and return provides a natural interpretation to the investment decisions. In this paper, the specific fuzzy membership functions are constituted for risk, return and beta separately. By using these fuzzy membership functions in the preemptive fuzzy programming approach, a novel portfolio selection model is developed. This model allows the decision makers not only to sort their fuzzy goals in the certain priorities, but also to determine the portfolios with respect to different types of strategies as risk-aversion, risk-neutral and risk seeking. In the application parts, the effectiveness of the proposed portfolio selection model is analyzed over the two investment periods having the upward and downward moving trend separately in the ISE30 index. According to the analysis results, the proposed approach gives better performance than Markowitz’s model in the both implementations too.

REFERENCE LIST


