MATRIX-BASED TIME/COST TRADE-OFF METHODS

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Abstract:
Due to the effects of the crisis, budgets of present as well as future projects are decreasing steadily. In this study a new exact method is introduced for minimising cost and time demands and combining time/cost trade-off methods and finding alternative project implementation techniques. This method supports not only the traditional but also the agile project management. Furthermore these methods can be used not only in case of network planning, but also for matrix-based project planning and solving trade-off methods. This study is a full paper version of the conference presentation, which was presented at the 30th Annual Hungarian Operation Research Conference.

Keywords: time/cost trade-off methods, cost minimizing, exact matrix-based project planning techniques

1. INTRODUCTION

In the course of planning and implementation of the projects it occurs frequently that after the preliminary planning of a project with optimal resources allocation to be implemented with minimal total costs cannot be realized at a price which is expected by the inviter of the tender. In this case traditional time/cost trade-off methods cannot find a feasible solution, because every project plan will be overbudgeted. Thus alternative project implementations should be defined. In this study a novel framework algorithm is introduced, where traditional time/cost trade-off methods and finding alternative project implementation techniques are combined based on matrix-based project network planning methods.

2. BACKGROUND OF THE STUDY – COST AND TIME MINIMISING METHODS

In these chapters different kinds of cost and time minimising and scheduling algorithm are shown. Most of them based on network planning techniques; however matrix-based methods can also be used for decreasing cost demands of the projects.

2.1. Time/cost and time/resource trade-off methods

The main proposal of the time/cost and the time/resource methods that there is a deterministic (Prabuddha et.al., 1995) or stochastic function (Feng, Liu & Burns, 2000) between duration time of activities and their cost or resource demands. Minimal project duration can be calculated in a very easy way: the project network should be scheduled as crash duration time (see left side of the Fig. 1) in case of every activity (see right side of the Fig. 1); however project costs are unnecessarily increased. Therefore in these methods two different kinds of target function can be defined:

1. Minimise total project time (TPT) or duration time of the project with minimal increase of cost and/or resources.
2. Minimise total project cost (TPC).

In this study only the first target function is considered. When using traditional project scheduling like CPM/PDM (see: Kelley & Walker, 1959; Roy, 1962) methods the scheduled duration times are considered as normal duration times of tasks. In this case minimal direct cost is assumed. Every decrease or increase of duration time causes increase of direct cost. There are minimal or crash duration times for every task. The cost/resource demands of crash durations are crash costs/resources (see the left side of the Fig. 1). The minimum value of the direct cost curve is in the normal duration time of the project. The decrease of the TPT infer the increase of direct cost. (More resources and advanced technologies are needed.) The minimal TPT is the bound of time reduction. Time-cost trade-off methods usually consider the interval normal and minimal TPT; however the delays of the project can also generate the increase of direct cost. While direct cost can increase in case of decreasing TPT, the indirect cost will decrease if the project can be completed shorter than the normal duration time (see the right side of the Fig. 1). Therefore time/cost trade-off methods can be used to determine minimal TPT and minimal TPC. When using this method the initial solution is the schedule result of CPM or PDM method, where TPT will be the normal duration time of the project. This problem can be solved by minimum cost flow (Ahuja, Magnati & Orlin, 1991), cost scaling (Röck, 1980; Goldberg & Tarjan, 1987), capacity scaling (Orlin, 1988; Plotkin & Tardos, 1990) algorithms. The running time of enhanced capacity scaling algorithm is: $O((m \log n)(m + n \log n))$, where m is the number of arcs in an activity on arc (AoA) project net, n is the number of nodes (events in a AoA net.)
According to time/cost and time/resource trade-off methods a new schedule of original project can be determined. In case of proposed new schedule, every task will be completed, but duration time of critical tasks will be decreased as much as possible. Unfortunately, against the cost minimising, the least total project cost may be higher than the planned project budget.

In this case there are three options: we give the implementation of the project up or we realize the project with losses or we replace some activities by new ones in order to reduce the costs (Kosztyán, Perjés & Bencsik, 2008).

In the course of our analysis we will not consider the first option any longer since in this case we lose this business and there is no sense in making any further optimal resource planning. The second option is sometimes undertaken when they estimate that in spite of the initial losses the deficit will return during the implementation of the subsequent projects. In this case the allocation of the resources which is optimal for the given target function involving a minimal total cost shall be determined.

Alternative implementation can mean alternative task completions (see i.e. (Feng, Liu & Burns, 1997; Kosztyán, Perjés & Bencsik, 2006), alternative project structures (see i.e. Kosztyán & Kiss, 2010), or alternative project scenarios (see i.e. Kosztyán & Kiss, 2011). Finding alternative implementation can be based on also matrix- and network-based project planning techniques; however it is shown that the matrix-based method can combine both time/cost trade-off and finding alternative implementation techniques.

2.2. Matrix-based project planning methods

Besides network planning techniques, matrix-based methods can also be used for project planning and scheduling problems. Matrix-based methods can describe the importance or probability values of task completions thus determining and ranking the importance or probability of possible project scenarios and project structures (Kosztyán & Kiss, 2010). By using matrix-based project planning methods the main challenge is to serve management claims.
Supporting the logic planning (Kosztyán & Kiss, 2010) matrix-based methods can be applied besides of traditional project planning techniques, aiming to meet management claims. Up to the present there were no fast and exact algorithms to select and order the first n possible (like first *n* least cost, least duration etc.) project plans. The under publication paper (Kosztyán, 2013) introduces new algorithms to select the best *n* piece (either most probable, shortest duration or lowest budgeted) project plans regarding (time/cost/resource) constraints.

Possible projects can be determined in two steps. First one should decide *which tasks should be executed*. Secondly, we should decide *how to execute* these project scenarios: parallel or sequential.
In that study four different kinds of scores are considered. These score values can be attached either to the project scenarios (noted by capital letters) or project structures (noted by lower case):

1. *Score value of task completion/task dependency* (*S,s*) describing either possible task completion or dependency. This value can mean
   a. Either probability value, where 1 means certain, between 0 and 1 means uncertain realisation
   b. or importance value of the task completion/dependencies, where 1 mean mandatory, between 0 and 1 means compulsory realisation.
2. *Score value of project scenario/project structure* (*P,p*) describing either possible project scenario or project structure. This value can mean
   a. Either probability value if score values of task completions/task dependencies are probability values. In this case the probability values of project scenario/project structure is the production of probability values of task completion/task dependencies.
   b. or importance value if score values of task completions/task dependencies are handled as importance values. In this case the importance values of project scenario/project structure are the sum of probability values of task completion/task dependencies.
3. *Score value of project duration time* (*T,t*)
4. *Score value of resource demands* (*R,r*)
5. *Score value of cost demands* (*C*)

If the most probable project plan cannot be finished within the time constraint; therefore, the next most probable project plan should be evaluated. If there is no feasible project plan within the most probable project scenario, the next most probable project scenario should be considered. This method called as EPR (Exact Project Ranking) method (see Kosztyán, 2013a; Kosztyán, 2013b). If the number of uncertain tasks is t and the number of uncertain dependencies is k then the most probable (most desired) project plan within a most probable (most desired) project scenario can be calculated within $O(k+t)$ step. The first n most probable (most desired) project plans can be specified within $O(n(k+t))$ step.

While this framework algorithm can find feasible, the most probable project scenarios, do not handle alternative task implementations, and do not integrate time/cost trade-off methods. The novelty of this study is that the proposed modified EPR algorithm can combine the alternative task implementation techniques and time/cost trade-off methods extending the consideration of uncertainty task completion and uncertainty task dependencies.

2.3. Extended Project Expert Matrix for handling alternative task completions and multilevel project planning problems

If completions of the tasks or (sub) projects are depending on each other than Boolean (and, or, exclusive or (xor) etc.) operators are used for characterizing the dependencies of task/subproject completions. This matrix-based approach is the *extended Project Expert Matrix*: xPEM (Kosztyán, 2012). Exclusive or (xor) operators can be used for representing alternative task completion (see Table 1).

Table 1: xPEM (extended Project Expert Matrix) – possible project scenarios and project plans ("x" denoted as exclusive or operation)

xPEM	A	B	C	D	Time	Cost
					Demands	
A	1	0,2	0,2	1	4 wks	50 000 €
B		0,6 x		1	3 wks	25 000 €
C			x 0,4	1	2 wks	20 000 €
D				0,3	1 wks	12 000 €

In this example 4 different kinds of project scenarios can be calculated, where either task ABD, ACD, AB or AC will be realised.

In this study different kinds of algorithms are combined in order to reduce time and/or cost demands.

3. COMBINING COST MINIMIZING METHODS

In the first step the representation of this problem will be shown. After that a modified framework algorithm will be introduced.

3.1. Problem representation

The modified extended Project Expert Matrix can represent the alternative task completions (see Table 1) and normal and crash demands (see Fig. 1: t_n normal, t_c crash duration times; $c(t_n)$ normal, $c(t_c)$ crash (direct) cost; $R(t_n)$ normal, $R(t_c)$ crash resource demands).

In this way both cost minimising and finding alternative project completion methods can be represented in the modified xPEM matrix. Score values of task completion/task dependency can mean either probability or importance values (see Fig. 2). If score values of task completion mean probability value than the score value of a probable task completion is the production of the probability of task completion, but if score values of task completions handled as importance values, than score value of a possible project scenario is the sum of importance values of task completions.

Score values of task completion and task dependencies can be defined, but if there is no a priori information dependencies/completions can be undefined. In this case undefined score values are handled as indifferent score values. It means we cannot order the project scenario or project structures by priorities or probabilities.

Fig. 1 shows a stochastic project plan. Since task B is a mandatory task completion (importance value is 1), the possible project scenarios are: A_1B, A_2B, A_3B, where the score value of possible project scenarios are the sum of the score values of task completion, therefore: $P_{A1B}=1.9>P_{A2B}=1.8>P_{A3B}=1.7$. If the first scenario will be selected, then the task completion will be sequential, while a second and third project scenario can be completed sequentially, but also in parallel. Parallel completion needs more resources but less time demands than sequential realises.

3.2. Specify the framework algorithm

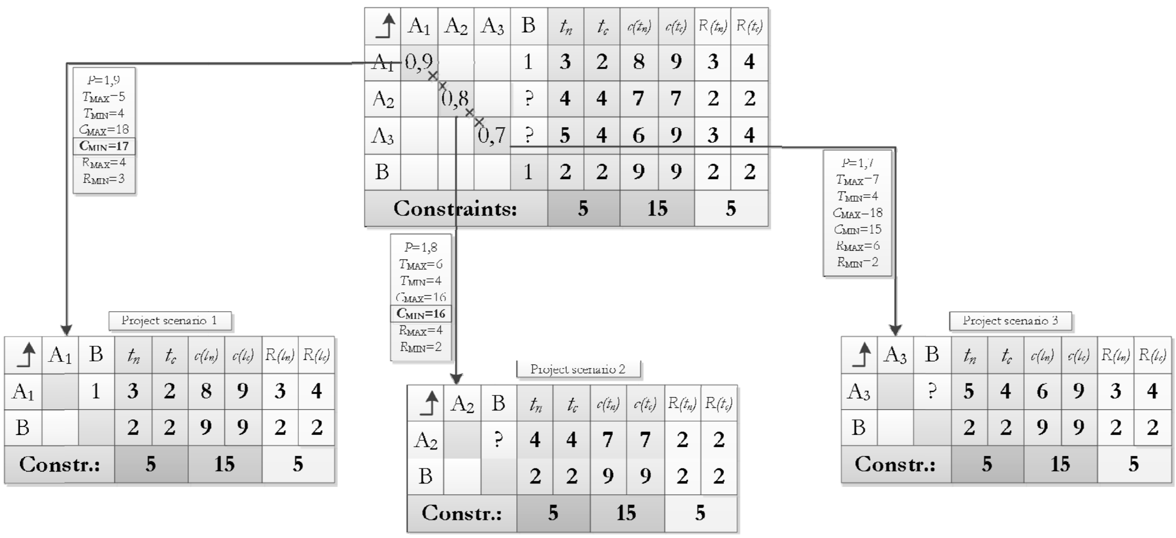
In the first step the minimal and maximal values of score values must be specified. This specification will be different from the formerly introduced (Kosztyán, 2013b) EPR algorithm, therefore this algorithm will be called *modified EPR algorithm*.

In this case the minimal/maximal importance value of project scenario will be the minimal/maximal sum of importance value of task completions. Minimal duration time (T_{MIN}) will occur, if every uncertain task completion and every uncertain dependency are ignored, and every duration time will be crash duration. Similarly maximal duration time (T_{MAX}) will occur, if every uncertain task completion and every uncertain dependency are realised, and every duration time will be normal duration. The maximal (direct) cost demand (C_{MAX}) will appear if every uncertain task are completed, and every task duration time is reduced to crash duration. Similarly the minimal (direct cost) (C_{MIN}) will occur if every uncertain task is ignored and every other task duration are normal duration. The maximal resource demands (R_{MIN}) will be minimal if every uncertain task completion is ignored, but every uncertain dependency will be realised and when calculating maximal resource demands the normal resource demands of the tasks are considered. In contrast, in case of as-soon-as possible scheduling, the maximal resource demands (R_{MAX}) will be maximal, if every uncertain task completion are realised, but every uncertain dependency will be ignored (see Fig. 1) and when calculating maximal resource demands the crash resource demands of the tasks are considered.

The second step is to specify constraints and the target function, where this function can be the selecting feasible most desired (most important) project scenario or less duration/less cost demanded project structures.

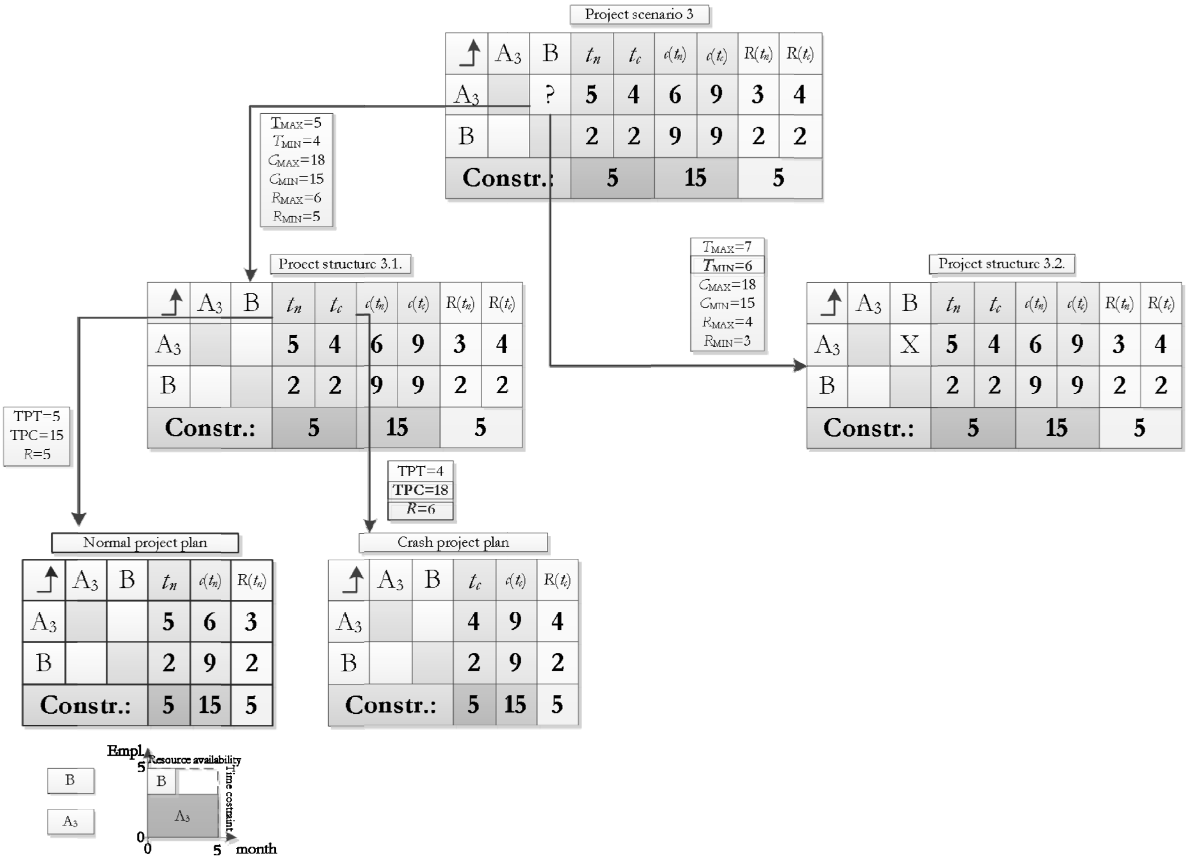
If a minimal score values of durations or direct costs are greater than the constraints, that this scenario is infeasible (see bordered, bold minimal score values in Fig. 1) and this scenario can be cut from the decision tree.

Picture 1: selecting feasible project scenario(s) (t_n normal, t_c crash duration times; $c(t_n)$ normal, $c(t_c)$ crash (direct) cost; $R(t_n)$ normal, $R(t_c)$ crash resource demands)



Similarly when finding feasible project structures, if a minimal score values of durations or direct costs are greater than the constraints, than this project structure is infeasible (see bordered, bold minimal score values in Fig. 1.) and this structure can be cut from the decision tree.

Picture 2: selecting feasible project structure(s) (t_n normal, t_c crash duration times; $c(t_n)$ normal, $c(t_c)$ crash (direct) cost; $R(t_n)$ normal, $R(t_c)$ crash resource demands; TPT=Total Project Time, TPC=Total Project Cost, "X" mean realised dependency)



Time/cost trade-off and resource allocation methods should be run after a project structure has been specified.

Using this modification the formerly introduced EPR framework algorithm can be applied. And the run time will be: $O(n(k+t)+nC)$, where n is the top n feasible project structure according to the given target function; t is the number of uncertain task completion, k is the number of uncertain task dependency and C is the run time of time/cost trade-off method.

4. SIMULATION RESULTS

In this chapter tests modified EPR algorithm in large, complex projects. 60 by 60 upper triangle matrices are generated, where every cell in upper triangle is randomised number between 0 and 1. The diagonal values mean uncertainty of task completion, where 0 means: these tasks will be ignored, 1 means: these tasks will be completed and between 0 and 1 means that these tasks either can be ignored or can be completed considering the time, cost and/or resource constraints. If a task completion is ignored, the score value of completion will be 0. If a task completion is decided to be realised, the score value of task completion will be considered. Let PEM be an n by n matrix, and $PEM_{ii} \in [0,1]$ i:=1..n the diagonal value of the PEM matrix. , −. 10 alternative tasks are specified, in the xPEM matrix, therefore 10 different kinds of 50 by 50 PEM matrix can be defined. Out of diagonal (score of dependencies) values can also fall between 0 and 1. In this simulation the score value of project structure has not been calculated, therefore considering only 3 cases are distinguished: p=1: task dependency will be realised (serial completion); p=0: task dependency will be ignored (parallel completion); $p \in (0,1)$: task dependency can be either realised or ignored, but the score value is indifferent.

Two kinds of durations: crash and normal duration; two kinds of cost demands: crash and normal cost demands of tasks; and for different kinds of resource demands are also randomised.

Time, cost and resource constraints are 80% and 90% of maximal score values of time, cost and resource demands. The target function is to specify 5 most desired feasible project plans. Instead of sum of score values of task completions and task dependencies, the mean values of these score values are represented, because if every score value of task completion and every score value of task dependency is between 0 and 1 and if these values follow uniform distribution the expected value of the mean of score values ( ) is 0.5. And the expected value of the score values of selected project scenarios and project plans are also between 0 and 0.5. In case of the most desired project plans

these values are maximal. Let is the *relative importance value of the project scenario j*. If P>0, then $P_j \in [0,1]$. P_j=1: if every task will be completed, and Pj=0: if every task is ignored from the project. Table 2 shows the simulation results based on 1000 generated 60 by 60 xPEM matrix with 10 different alternative tasks.

Table 2: The results of simulation, where every cell represents a mean value of relative importance value of a project scenario based on 1000 simulation.

Mean score values of top n most desired feasible project scenarios	Budget/Maximal Cost Demands							
	80%				90%			
	Resource Availability / Maximal Resource Demands							
	80%		90%		80%		90%	
	Time Constraint / Maximal Time Demands							
	80%	90%	80%	90%	80%	90%	80%	90%
P_1	0.80	0.82	0.84	0.85	0.95	0.95	0.96	0.97
P_2	0.76	0.72	0.71	0.75	0.95	0.95	0.96	0.96
P_3	0.74	0.72	0.70	0.74	0.94	0.93	0.93	0.91
P_4	0.71	0.71	0.70	0.73	0.91	0.91	0.91	0.90
P_5	0.71	0.71	0.70	0.73	0.90	0.90	0.91	0.90

With using multivariate analysis of variance (MANOVA) and decision tree techniques, the most significant constraint for changing relative importance value of a scenario is the budget (or cost constraint). Every relative importance value of the most desired project scenario is not lower than the budget/maximal cost demands, even if the relative time constraint (time constraint/maximal time demands) and relative resource constraints (resource availability/maximal resource constraint) are 90% or 80%. Time constraints can also be handled by parallelisation (ignoring task dependencies) and resource constraint can also be handled by sequencing (realising task dependencies). While this problem can be handled if these problems are separated, if we decrease these constraints by using time/cost trade-off methods and finding alternative solution techniques uncertain task also should be ignored in order to find feasible solutions.

5. SUMMARY AND CONCLUSION

In this paper a modified matrix-based project planning method was introduced. In the course of the evaluation three matrix-based methods are used: PEM for characterizing multilevel projects; SNPM for describing a project scenario and a DSM for presenting a project structure. Both cost minimising and finding alternative solution methods are combined into this method. Since there are huge number of variations of different kinds of project scenarios and project structures, an exact algorithm should be used for selecting adequate project scenarios and project structures considering the management claims.

The modified Expert Project Ranking (EPR) method is a fast, exact method for selecting and ranking feasible project scenarios and project plans within different kinds of project scenarios. Applying score values the most desired, most probable feasible project plans can be selected considering time, cost and resource constraints.

Although the project expert system seems rather fictitious currently, the developed matrix based methods and proposed exact algorithms may be important and essential components of a project expert system supporting strategic decision makings.

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