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BASIC QUATERNARY DESIGN TABLE USING GEOMETRICAL DESIGN

Jen-der Day, Department of Industrial Engineering and Management,
National Kaohsiung University of Applied Sciences, Kaohsiung, TAIWAN.
jdd@kuas.edu.tw

Hsien-Tang Tsai, Department of Business Management,
National Sun Yen-sen University, Kaohsiung, TAIWAN.
htt@mail.nsysu.edu.tw

ABSTRACT

Purpose *Taguchi provided some useful tools such as various orthogonal arrays, interaction tables, linear graphs, etc. for planning fractional factorial experiments and had many successful application cases in quality engineering (Taguchi, 1986). However, many research articles explored the methods that were used to construct those tools and tried to improve them. The aim of this article is to develop a new tool to substitute the uses of Taguchi's orthogonal arrays and interaction tables.*
Design/methodology/approach *Using a number representation system whose base is a power of 2, Tsai (1999) developed an easy algorithm for obtaining multi-factor interaction columns in geometrical designs, which serves as theoretic background for the development of a new tool.*
Finding *Based on the algorithm of base 4, in this article we propose a Basic Quaternary Design Table (BQDT) which is a 4 by 4 squared matrix with entries of both decimal and quaternary column numbers. A BQDT has a nice structure of confounding relationships so that users could identify multi-factor interaction columns in a straightforward manner without looking up tables. The advantages of the proposed BQDT include (1) it serves as an efficient tool for column assignment problem; (2) it can substitute the use of Taguchi's interaction table; (3) it is visually appealing such that the users can easily recognize some special designs.*
Originality/value *Both geometrical design matrix and the BQDT can be used jointly to plan a two-level fractional factorial experiment without looking up tables, which can substitute the uses of Taguchi's orthogonal arrays and interaction tables when run size n is a power of 2.*

Key words: Geometrical design, Quaternary, Taguchi's interaction table
Classification: Research paper
Subject Area: Quality improvement and management

INTRODUCTION

Taguchi's orthogonal arrays are useful tool for planning fractional factorial experiments and have many successful application cases in quality engineering (Taguchi, 1986). For planning an experiment, Taguchi provided some useful tools such as various orthogonal arrays (or design matrices), interaction tables, linear graphs, etc. and adopted the column assignment method in which the required factors are assigned to appropriate columns of a given orthogonal array. However, many research articles explored the methods that were used to construct those tools and tried to improve them (Bullington, et al., 1990; Kacker, et al., 1991).

The geometrical design (G_n) proposed by Plackett & Burman (1946), which is the same as a regular Hadamard matrix when the run size of n is a power of 2, can be constructed easily by using a consecutively doubling method without any computation or looking up tables; that has a practical advantage for uses and overwhelms the use of Taguchi's orthogonal arrays. Taguchi's interaction table is a n by n triangle matrix with the entries of two-factor interaction columns, where n is the number of run size of an orthogonal array. The table can be used to find multi-factor interaction columns efficiently; however, its size becomes larger as n increases so as to create a big burden for uses by looking up lager tables.

The aim of this article is to develop a new efficient tool based on geometrical designs to substitute the uses of Taguchi's orthogonal arrays and interaction tables. Using a number representation system whose base is a power of 2, Tsai (1999) developed an easy algorithm for obtaining two-factor or multi-factor interaction column in geometrical designs. Based on the results of base 4, a Basic Quaternary Design Table (BQDT) is proposed in this article for planning two-level fractional factorial experiments. The content is organized as follows: geometrical designs and the number representation algorithm for 2fi are stated first, then a BQDT is proposed and its wordlength pattern is discussed, finally some special designs using the BSDT with visually appealing are presented and discussed.

GEOMETRICAL DESIGNS

A doubling method was given by Plackett and Burman (1946, p313): If A is orthogonal,

$\mathbf{B}=\begin{bmatrix}\mathbf{A} & \mathbf{A}\\ \mathbf{A} & -\mathbf{A}\end{bmatrix}$ is also orthogonal and has double the order of A. Note that B can be expressed as B=[A_L, A_R], where A_L=[A, A]', A_R=[A, -A]', and A_R is called "fold-over" by Box and Wilson (1951). Starting from G_2, geometrical designs (GD's) can be obtained by the successive doubling method as follows:

$$\mathbf{G_2}=\begin{bmatrix}\mathbf{1} & \mathbf{1}\\ \mathbf{1} & \mathbf{-1}\end{bmatrix},\ \mathbf{G_4}=\begin{bmatrix}\mathbf{G_2} & \mathbf{G_2}\\ \mathbf{G_2} & -\mathbf{G_2}\end{bmatrix}=\begin{bmatrix}\mathbf{1} & \mathbf{1} & \mathbf{1} & \mathbf{1}\\ \mathbf{1} & \mathbf{-1} & \mathbf{1} & \mathbf{-1}\\ \mathbf{1} & \mathbf{1} & \mathbf{-1} & \mathbf{-1}\\ \mathbf{1} & \mathbf{-1} & \mathbf{-1} & \mathbf{1}\end{bmatrix}, \text{ and}$$

$$\mathbf{G_8}=\begin{bmatrix}\mathbf{G_4} & \mathbf{G_4}\\ \mathbf{G_4} & -\mathbf{G_4}\end{bmatrix}=\begin{bmatrix}1&1&1&1&1&1&1&1\\1&-1&1&-1&1&-1&1&-1\\1&1&-1&-1&1&1&-1&-1\\1&-1&-1&1&1&-1&-1&1\\1&1&1&1&-1&-1&-1&-1\\1&-1&1&-1&-1&1&-1&1\\1&1&-1&-1&-1&-1&1&1\\1&-1&-1&1&-1&1&1&-1\end{bmatrix}.$$

Similarly, all geometrical designs with higher orders such as G_{16}, G_{32}, G_{64}, etc. could be obtained easily by hand writing, or by using "Excel" functions such as "copy" and "replace" without any computation. Note that a geometrical design is the same as Hadamard matrix when its run size is a power of 2.

NR ALGORITHM FOR 2FI IN GD'S

Tsai (1999) showed that the doubling method has a nice recursive property for obtaining two-factor interactions (2fi's). By observing the basic matrix of doubling $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$, the 2fi of $\begin{bmatrix}1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\1\end{bmatrix}$ is $\begin{bmatrix}1\\1\end{bmatrix}$; $\begin{bmatrix}1\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\-1\end{bmatrix}$ is $\begin{bmatrix}1\\1\end{bmatrix}$; $\begin{bmatrix}1\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\1\end{bmatrix}$ is $\begin{bmatrix}1\\-1\end{bmatrix}$, respectively. This property is well preserved during the process of doubling. If a geometrical design is partitioned as left-half and right-half parts such as $G_{2n}=[A_L, A_R]$, where $A_L=[G_n, G_n]'$, $A_R=[G_n, -G_n]'$, then three confounding relationships can be observed as follows:

(1) The 2fi column of any two given columns in A_L remains in A_L.
(2) The 2fi column of any two given columns in A_R remains in A_L.
(3) The 2fi column of any two given columns of which one is in A_L and the other one is in A_R remains in A_R.

Namely, the basic rule for 2fi is "left by left → left"; "right by right → left"; "left by right →right", or in short A_L x A_L → A_L; A_R x A_R → A_L; A_L x A_R→ A_R. This rule can be easily extended to higher order interactions.

Based on a number representation system whose base is a power of 2, Tsai (1999) proposed an efficient method, called NR method, for obtaining the 2fi column. Let E denote the 2fi column of two given columns C and D, namely, $E=T_n(C,D)$, then the NR method for quaternary case is stated as below:

1. Convert two given column numbers C and D into digits with base 4.
2. Compute $E=T_n(C,D)$ digit by digit according to the basic confounding relationships in G_4 such as $T_4(1,2)=3$; $T_4(1,3)=2$; $T_4(2,3)=1$; $T_4(0,j)=j$; $T_4(j,j)=0$, j=0,1,2,3.
3. Convert the resulting digit numbers with base 4 back to a decimal digit number.

Two numerical examples are given below:

a. $T_{16}(9,14)=T_{16}([21,32])=[13]=7$;
b. $T_{128}(13,99)=T_{128}([0031],[1203])=[1232]=110$.

BASIC QUATERNARY DESIGN TABLE

Based on the above NR algorithm for the quaternary case, since its basic confounding relationship $T_4(i,j)$, i, j=0,1,2,3 is quiet straightforward and is almost memory free, we can easily construct a “Basic Quaternary Design Table (BQDT)” as in Table 1.

Table 1. A Basic Quaternary Design Table (BQDT) for G_{16}.

0 (00)	4 (10)	8 (20)	12 (30)
1 (01)	5 (11)	9 (21)	13 (31)
2 (02)	6 (12)	10 (22)	14 (32)
3 (03)	7 (13)	11 (23)	15 (33)

This is a two-level geometrical design with 16 runs, say G_{16}, in which all sixteen columns are arranged in a 4x4 square table, the entries of 0 to 15 are decimal column numbers while the entries in the parentheses are corresponding quaternary column numbers. The usefulness of this BQDT has two aspects: (1) the required factors can be assigned directly to corresponding columns to form a fractional factorial design; note that the column 0 is an identity column which cannot be assigned any factor; (2) any multi-factors interaction can be obtained naturally according to the basic confounding relationships in G_4, for example, the 2fi of column 9 (21) and column 14 (32) is column 7 (13). Therefore, applying the basic confounding relationships in G_4, we can obtain the 2fi of any two given columns in a straightforward manner without looking up any interaction table. In other words, the BQDT can substitute the use of Taguchi’s interaction table in a more natural way.

WORDLENGTH PATTERN OF BQDT

Maximum resolution (Box & Hunter, 1961) and minimum aberration (Fries & Hunter, 1980) are two important criteria for choosing a good fractional factorial design, in which the wordlength patterns (WLP's) are required for comparisons. A defining relation is a word of letters (factors) denoted by 1, 2, 3…(or A, B, C…) and the number of letters in a word is called as its wordlength. A 2^{n-p} fractional factorial design d with n factors is uniquely determined by p independent defining relations (words) which generate the defining contrast subgroup. Then, the wordlength pattern of a design d is defined as the vector WLP(d)=[A_1(d), A_2(d), …, A_n(d)], where A_k(d) is the number of length-k words in the defining contrast subgroup. In this section, we will discuss the most often used length-four and length-three words, respectively.

Assume that 15 factors are assigned to columns 1-15 in a saturated 2^{15-11} design and a BQDT (G_{16}) is used to obtain both A_3 and A_4. To facilitate the presentation, let j-th column set be denoted by C(j) = {(j,0), (j,1), (j,2), (j,3)} while i-th column set is denoted by C(i) = {(0,i), (1,i), (2,i), (3,i)}. By using combinatorial method, all four-factor interactions (4fi’s) confounded with the identity column (column 0) will be identified first, and they are classified into five different categories. Note that column 0 is also assumed as one factor and is included in the analysis.

(a) Column and row: There are 8 4fi's in three column sets and three row sets, for examples, T_4(10,11,12,13)=00; T_4(03,13,23,33)=00.

(b) Pair in each of two column sets: Consider a pair factors are in each of two column sets, for example, in column sets 0 and 1, (00,01,12,13), (10,11,02,03), (00,11,02,13), (10,01,12,03), (00,11,12,03), (10,01,02,13) confounded with column 0. There are $\binom{4}{2}=6$ combinations for four column sets, and 6 4fic's are in each combination, therefore 6x6=36 4fic's are obtained in subtotal.

(c) Pair in each of two row sets: Similar to column sets above, 36 4fic's are obtained in subtotal. For example, in row sets 0 and 1, (00,10,21,31), (01,11,20,30), (00,11,20,31), (01,10,21,30), (00,11,21,30), (01,10,20,31) confounded with column 0.

(d) 2x2 Square: There are 6 2x2 square in any two column sets, for example, in column set 0 and 1, (00,01,10,11), (00,02,10,12), (00,03,10,13), (01,02,11,12), (01,03,11,13), (02,03,12,13). Thus, there are 6 combinations for four columns sets, and 6 4fic's are in each combination, therefore 6x6=36 4fic's are obtained in subtotal.

(e) Latin Square: Similar to a Latin Square, each factor can only be shown up once either in a row or a column, example, (00,11,22,33),(01,10,23,32), (02,10,21,33), etc. There are 4!=24 4fic's in a 4x4 Latin Square.

In total, there are 8+36+36+36+24=140 4fi's in a BQDT with 16 factors including column 0. If a four-factor interaction (4fi) contains the identity column (column 0) then it is a length-3 word, otherwise it is a length-4 word. By fixing column 0 in five categories, the number of length-three words is equal to A_3=2+9+9+9+6=35; while the number of length-four words is equal to A_4=6+27+27+27+18=105.

SOME SPECIAL DESIGNS

Chen, Sun and Wu (1993) provided a catalogue of two-level and three-level fractional factorial designs with small runs, the additional column numbers other than independent columns are reported and the design numbers are arranged as [k-p.i] in the given tables. Two special designs will be adopted to illustrate the use of a BQDT, and how to construct other alternative designs. For a BQDT with run size n=16, the left-half part contains columns 0-7 while the right-half part contains 8-15.

(a) Even MA Resolution IV designs

The series of $\mathbf{2}_{\mathbf{IV}}^{\mathbf{4-1}}, \mathbf{2}_{\mathbf{IV}}^{\mathbf{8-3}}, \mathbf{2}_{\mathbf{IV}}^{\mathbf{16-11}}, \mathbf{2}_{\mathbf{IV}}^{\mathbf{32-26}}$,... has twice as many runs (n=2k) as factors (k) and are even minimum aberration (MV) resolution IV designs, in which each of k-1 alias sets contains k/2 2fi's. Table 2 shows an example of k=8 in [8-4.1], factors are assigned to columns {1,2,4,5,8,11,13,14}, each of 7 blank columns contains 4 2fi's, and W=(0,14,0,0,0,1). Table 3 shows another similar even MA resolution design in which factors are assigned to columns 8-15 in the right-half part, each of 7 blank columns in the left-half part contains 4 2fi's, and with the same W=(0,14,0,0,0,1).

Table 2. The design of 2^{8-4} with even MA resolution IV in [8-4.1].

00	10	20	
01			31
02			32
	13	23	

Table 3. The similar design of 2^{8-4} with even MA resolution IV.

00		20	30
		21	31
		22	32
		23	33

(b) Maximal number of Clear 2fi's

In Wu and Chen (1992), any two-factor interaction (2fi) that is not aliased with any main effect or other 2fi's is called "clear". Table 4 shows that 8 factors are assigned to columns {1,2,3,4,5,6,7,8} in [8-4.6], the 2fi of any two factors in left-half part remains in left-half part; the 2fi of column 8 with any column in left-half part remains in right-half part. Obviously, seven clear 2fi's in columns 9-15 are clear. Table 5 shows that if column 8 is replaced by any one from columns 9-15 in Table 10, then the design also have seven clear 2fi's in right-half part.

Table 4. The design of 2^{8-4} with seven clear 2fi's in [8-4.6].

00	10	20	
01	11		
02	12		
03	13		

Table 5. The similar design of 2^{8-4} with seven clear 2fi's.

00	10		30
01	11		
02	12		
03	13		

Above all, Tables 2, 3, 4, and 5 show that the BQDT is visually appealing in which some special patterns and confounding relationships exist for special designs. This highlights another advantage of BQDT over the Taguchi's interaction tables.

CONCLUSIONS

A Basic Quaternary Design Table (BQDT) was proposed as a new tool for an experimenter to plan a two-level fractional factorial design, in which all required factors can be assigned directly into the corresponding columns of the table. The BQDT has a nice structure of confounding relationships so that users could identify any interaction columns in a straightforward manner without looking up tables. The length-three or length-four words of the BQDT are classified into five categories. Besides, some special designs such as even minimum aberration resolution IV or maximal clear 2fi's could be constructed easily with the help of a BQDT. Note that one of the key advantages to the practical users is that the BQDT is visually appealing. Furthermore, the BQDT can be extended to a large case in the future research.

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