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FORECASTING WITH FOURIER RESIDUAL MODIFIED ARIMA MODEL- THE CASE OF AIR CARGO IN TAIWAN

Thanh-Lam Nguyen, Graduate Institute of Mechanical and Precision Engineering
National Kaohsiung University of Applied Sciences, Kaohsiung 80778, Taiwan

Peng-Jen Chen, Department of Industrial Engineering and Management
National Kaohsiung University of Applied Sciences, Kaohsiung 80778, Taiwan

Ming-Hung Shu, Department of Industrial Engineering and Management
National Kaohsiung University of Applied Sciences, Kaohsiung 80778, Taiwan

Bi-Min Hsu, Department of Industrial Engineering and Management
Cheng Shiu University, Kaohsiung 83347, Taiwan

Yi-Chang Lai, Department of Industrial Engineering and Management
National Kaohsiung University of Applied Sciences, Kaohsiung 80778, Taiwan

ABSTRACT

Purpose- *Import-export activities normally require the involvement of different sectors needing proper plans to make the trade-flow stably grown. This paper is aimed at establishing an accurate forecasting model for trade volume.*
Design/methodology/approach- *In this study, a new forecasting model named ARIMAF/ SARIMAF was proposed by combining the Fourier series with the conventional ARIMA/ SARIMA forecasting model.*
Findings- *In the cases of historical data of the volumes of imported and exported air cargo in Taiwan,* $SARIMAF(3,1,1)(1,1,1)_{12}$ *and* $SARIMAF(2,1,3)(1,1,1)_{12}$ *models were found fitting well with the mean absolute percentage error values of 0.0104 and 0.0096, respectively. Using these two models, the monthly volumes of air cargo in Taiwan in 2012 are predicted.*
Originality/value- *The using Fourier series to modify the residuals of the traditional ARIMA is first proposed in this paper.*

Keywords: Fourier residual modification, ARIMA, forecasting air cargo volume

INTRODUCTION

International trade is defined as the exchange of goods, capital and services across national borders or territories, which usually takes the most important share in the gross domestic products in many countries around the world. It has become one of the most critical factors to develop the national economies and the nations themselves, because most of their resources need rearranged and reallocated for better performance through export activities; also, the countries could have what they need for their people as well as for building their internal powers through import activities. Without international trade, the goods and services available in a nation are limited to what can be produced within its border; some certain goods/services become superfluous if the nation has comparative advantages, whereas, some turn into scarce for consumption and development.

Import and export, two basic activities in international trade, normally require the involvement of many different sectors from purchasing, manufacturing, inventory, transportation, distribution, etc., especially the engagement of the customs authorities in both countries. Therefore, it is of great significance to have appropriate planning from macro to micro levels so that the trade flow can develop affluently among nations. In order to have proper plans, an accurate forecast of the volume of imported-exported goods becomes compulsory.

In predicting the volume of containers used in international trade, many researchers used conventional regression methods only which disregard non-stationary relationship among the volume of containers and the macro-economic variables possibly resulting in the spurious regression forecasting models (Chou et al, 2008).

Autoregressive Integrated Moving Average (ARIMA) model is one of well-known models dealing with time series. And in order to enhance the accuracy level of this model, in this study, a new approach to minimize its residuals is suggested by modifying them with Fourier series. This modification results in a new model, hereinafter called ARIMAF. An empirical study of air cargo in Taiwan is conducted to compare the accuracy of the two mentioned models so as to find out an appropriate one for forecasting the volume of imported and exported air cargo in Taiwan.

LITERATURE REVIEW

1. ARIMA Model

ARIMA model was first introduced by Box and Jenkins in 1960s to forecast a time series which can be made stationary by differencing or logging. A time series may have non-seasonal or seasonal characteristics. Seasonality in a time series is defined as a regular pattern of changes that repeats over S time-periods. With a seasonal time series, there is usually a difference between the average values at some particular times within the seasonal intervals and the average values at other times; therefore, in most cases, a seasonal time series is non-stationary. With a seasonal time series, it can be made stationary by seasonal differencing which is defined as a difference between a value and a value with lag that is a multiple of S.
A non-seasonal ARIMA model has the form of ARIMA(p,d,q) while the seasonal ARIMA model comes in the form of SARIMA(p,d,q)$(P,D,Q)_S$, where:

- p is Auto-Regressive term.
- d is Integrated term.
- q is Moving-Average term.
- P is the number of seasonal Autoregressive order.
- D is the number of seasonal differencing.
- Q is the number of seasonal Moving Average order.
- S is the time span of repeating seasonal pattern.

“Auto-Regressive” term refers to the lags of the differenced series appeared in the forecasting equation and “Moving Average” term refers to the lags of the forecast errors. A time series differenced to be made stationary is said to be "integrated". “Integrated” term refers to the difference levels to make a time series stationary.

The overall procedures of ARIMA (SARIMA) methodology are: (1) identifying the possible models; (2) fitting the models; and (3) testing for their adequacy & reliability before being applied to forecast (Hanke & Wichern, 2005).

** Step 1: Identifying the possible models*

- By using the Auto-Correlation Function (ACF) graph, the stationarity of the time series can be easily determined. If the lag values of the graph cut off fairly quickly or die down fairly quickly, the time series is considered stationary; otherwise, it isn't. If the series is not stationary, it should be differenced gradually until it is considered stationary. Then, the d value in the ARIMA(p,d,q) is obtained. The values of p and q are determined based on ACF and Partial Auto-Correlation Function (PACF) graph as in table 1.

Table 1: Determine the p & q parameters by using ACF and PACF graph

Model	**ACF**	**PACF**
AR(p)	Dies down fairly quickly	Cut off after lags p fairly quickly
MA(q)	Cut off after lags q fairly quickly	Dies down fairly quickly
ARMA(p,q)	Dies down fairly quickly	Dies down fairly quickly

- Examine the patterns across lags that are multiples of S to identify seasonal terms. Judge the ACF and PACF at the seasonal lags in the same way.

** Step 2: Fitting the model*

Once a tentative model is selected, its parameters are then estimated. Therefore, this stage is also called "Estimating the model".

** Step 3: Testing the model for adequacy*

The model adequacy is tested by considering the properties of the residuals. The residuals must have normal distribution and be white-noise (also known random). This test can be done with one of the following ways:

- Ljung-Box Q statistic:

$$Q_m = n(n+2)\sum_{k=1}^{m}\frac{e_k^2}{n-k} \tag{1}$$

where: e_k is the residual autocorrelation at lag k
n is the number of residuals
m is the number of time lags includes in the test

If the p-value associated with the Ljung-Box Q Statistic is smaller than a given significance, the model is considered inadequate. Otherwise, it is appropriate for further application.

- Testing the normal distribution of the residuals by considering the normal probability plot and testing the white-noise of the residuals by considering its ACF and PACF graphs where individual residual autocorrelation should be small and its value is within $\pm 2/\sqrt{n}$ from the central point of zero.

2. Fourier Residual Modification ARIMA Model (ARIMAF)

In order to improve the accuracy of forecasting models, the Fourier series has been successfully applied in modifying the residuals in Grey forecasting model GM(1,1) (Askari & Fetanat, 2011; Guo et al, 2005; Hsu, 2003; Huang & Lee, 2011; Kan et al, 2010) which reduces the values of RMSE, MAE, MAPE, etc., Thus, this good methodology should be considered in the case of ARIMA (SARIMA) model.

The procedure to obtain the modified residuals from ARIMA (SARIMA) model with Fourier series is as the following.

Suppose an original series with n entries is $x = \{x(1), x(2), x(3), \ldots, x(k), \ldots, x(n)\}$ and its predicted series under ARIMA (SARIMA) is $\hat{x} = \{\hat{x}(1), \hat{x}(2), \hat{x}(3), \ldots, \hat{x}(k), \ldots, \hat{x}(n)\}$; then, its residual series ε is defined as:

$$\varepsilon = \{\varepsilon(1), \varepsilon(2), \varepsilon(3), \ldots, \varepsilon(k), \ldots, \varepsilon(n)\} \qquad \left(k = \overline{1,n}\right) \tag{1}$$

where $\varepsilon(k) = x(k) - \hat{x}(k) \qquad \left(k = \overline{1,n}\right)$

Now, let's consider a sub-series ε^* as:

$$\varepsilon^* = \{\varepsilon(2), \varepsilon(3), \ldots, \varepsilon(k), \ldots, \varepsilon(n)\} \qquad \left(k = \overline{2,n}\right) \tag{2}$$

Expressed in Fourier series, $\varepsilon(k)$ is rewritten as:

$$\varepsilon(k) = \frac{1}{2}a_0 + \sum_{i=1}^{D}\left[a_i \cos\left(\frac{2\pi i}{n-1}k\right) + b_i \sin\left(\frac{2\pi i}{n-1}k\right)\right] \qquad \left(k = \overline{2,n}\right) \tag{3}$$

where $D = \left[(n-1)/2 - 1\right]$ is called the minimum deployment frequency of Fourier series (Huang & Lee, 2011) and only take integer number (Askari & Fetanat, 2011; Guo et al, 2005; Hsu, 2003).

And therefore, the residual sub-series ε^* is rewritten as:

$$\varepsilon^* = P.C \tag{4}$$

where

$$P = \begin{bmatrix} \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times 2\right) & \cdots & \cos\left(\frac{2\pi\times D}{n-1}\times 2\right) & \sin\left(\frac{2\pi\times D}{n-1}\times 2\right) \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times 3\right) & \cdots & \cos\left(\frac{2\pi\times D}{n-1}\times 3\right) & \sin\left(\frac{2\pi\times D}{n-1}\times 3\right) \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \frac{1}{2} & \cos\left(\frac{2\pi\times 1}{n-1}\times n\right) & \sin\left(\frac{2\pi\times 1}{n-1}\times n\right) & \cdots & \cos\left(\frac{2\pi\times D}{n-1}\times n\right) & \sin\left(\frac{2\pi\times D}{n-1}\times n\right) \end{bmatrix}$$

$$C = \left[a_0, a_1, b_1, a_2, b_2, \ldots, a_D, b_D\right]^T$$

By using the ordinary least squares method (OLS), the values of $a_0, a_1, b_1, a_2, b_2, \ldots, a_D, b_D$ are obtained from the equation of:

$$C = \left(P^T P\right)^{-1} P^T \left[\varepsilon *\right]^T \tag{5}$$

Then, the predicted residual $\hat{\varepsilon}(k)$ is now easily achieved based on the equation (3) as:

$$\hat{\varepsilon}(k) = \frac{1}{2}a_0 + \sum_{i=1}^{D}\left[a_i \cos\left(\frac{2\pi i}{n-1}k\right) + b_i \sin\left(\frac{2\pi i}{n-1}k\right)\right] \tag{6}$$

So, based on the predicted series $\hat{x}$ obtained previously, the predicted series $\breve{x}$ from the Fourier residual modification of ARIMA model (SARIMA) is determined by:

$$\breve{x} = \{\breve{x}(1), \breve{x}(2), \breve{x}(3), \ldots, \breve{x}(k), \ldots, \breve{x}(n)\} \tag{7}$$

where $$\begin{cases} \breve{x}(1) = \hat{x}(1) \\ \breve{x}(k) = \hat{x}(k) + \hat{\varepsilon}(k) \qquad \left(k = \overline{2,n}\right) \end{cases}$$

In evaluating the model accuracy and comparing models, there are important indexes to be considered, such as:

- The mean absolute percentage error (MAPE) (Askari & Fetanat, 2011; Chang & Liao, 2010; Guo et al, 2005; Hsu & Chen, 2003; Hsu & Wang, 2008; Huang & Lee, 2011; Kan et al, 2010; Li et al, 2011; Tsaur & Kuo, 2011):

$$MAPE = \frac{1}{n}\sum_{k=1}^{n}\frac{|x(k) - v(k)|}{x(k)} \qquad \left(k = \overline{1,n}\right)$$

where $v(k)$ is the predicted value of entry k $\left(k = \overline{1,n}\right)$ (so, $v(k) = \hat{x}(k)$ in ARIMA model or $v(k) = \breve{x}(k)$ in SARIMA model).

+ The post-error ratio C (Hua & Liang, 2009; Ma & Zhang, 2009): $C = \frac{S_2}{S_1}$

where: + $S_1 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}[x(k) - \bar{x}]^2}$ *where* $\bar{x} = \frac{1}{n}\sum_{k=1}^{n} x(k)$

+ $S_2 = \sqrt{\frac{1}{n}\sum_{k=1}^{n}[\varepsilon(k) - \bar{\varepsilon}]^2}$ *where* $\varepsilon(k) = x(k) - v(k)$ *and* $\bar{\varepsilon} = \frac{1}{n}\sum_{k=1}^{n}\varepsilon(k)$

The smaller the C value is, the higher accuracy the model has since smaller C value results from a larger S_1 and/or a smaller S_2.

- The small error probability P (Hua & Liang, 2009; Ma & Zhang, 2009):

$$P = p\left\{\frac{|\varepsilon(k) - \bar{\varepsilon}|}{S_1} < 0.6745\right\}$$

The higher the P value is, the higher accuracy the model has since P value indicates the probability of the ratio of the difference between the residual values of data points and the average residual value with the standard deviation of the original series smaller than 0.6745 (Ma & Zhang, 2009).

- The forecasting accuracy ρ (Ma & Zhang, 2009): $\rho = 1 - MAPE$

Based on the above indexes, there are four grades of forecasting accuracy as stated in table 1.

Table 1: Four grades of forecasting accuracy

Grade level	*MAPE*	*C*	*P*	ρ
I (Very good)	< 0.01	< 0.35	> 0.95	> 0.95
II (Good)	< 0.05	< 0.50	> 0.80	> 0.90
III (Qualified)	< 0.10	< 0.65	> 0.70	> 0.85
IV (Unqualified)	≥ 0.10	≥ 0.65	≤ 0.70	≤ 0.85

METHODS

The historical data of the volume of imported-exported air cargo in Taiwan are obtained from the monthly statistical data published on the website of Ministry of Transportation and Communication R.O.C (MOTC) from January 2002 to December 2011. There are totally 120 observations of monthly volume of imported air cargo and exported air cargo of Taiwan.

The data are used to establish relevant ARIMA (SARIMA) and ARIMAF (SARIMAF) models which are then tested for their accuracy before they are used to predict the volume of imported-exported air cargo in Taiwan in year 2012. The volumes of imported and exported air cargo are respectively named as IMP and EXP variables.

To deal with ARIMA (SARIMA) model, computer software called Statistical Package for the Social Sciences (SPSS) is used whereas Microsoft Excel is used to deal with the Fourier residual modification.

Firstly, using SPSS program, time series plots are examined for the seasonality feature before coming up with ARIMA (SARIMA) models. After an appropriate model is selected, it is then used to calculate the predicted values of the observations to figure out its accuracy level and its residual series as well.

Secondly, in order to increase the accuracy level of the ARIMA (SARIMA) model, the residual series is modified with Fourier series as expressed in section 2.2. At the end of this stage, a modified residual series is obtained for the next step.

Thirdly, with the modified series, a predicted series under ARIMAF (SARIMAF) model is calculated as stated in equation (7). This predicted series will be used to determine the actual residuals from ARIMAF (SARIMAF) model for evaluating its accuracy.

Finally, a comparison between the conventional ARIMA (SARIMA) model and ARIMAF (SARIMAF) model is conducted to prove that ARIMAF (SARIMAF) model is better than ARIMA (SARIMA) model.

EMPIRICAL STUDY

1. The Volume of Imported Air Cargo in Taiwan

1.1 ARIMA model

The data of the volume of imported air cargo are tested and show that the original data set of IMP is not stationary but seasonal. However, at one degree of both non-seasonal and seasonal difference, the series becomes stationary. Figure 1 shows that there are two possible SARIMA models for IMP: $SARIMA(1,1,1)(1,1,1)_{12}$ and $SARIMA(3,1,1)(1,1,1)_{12}$.

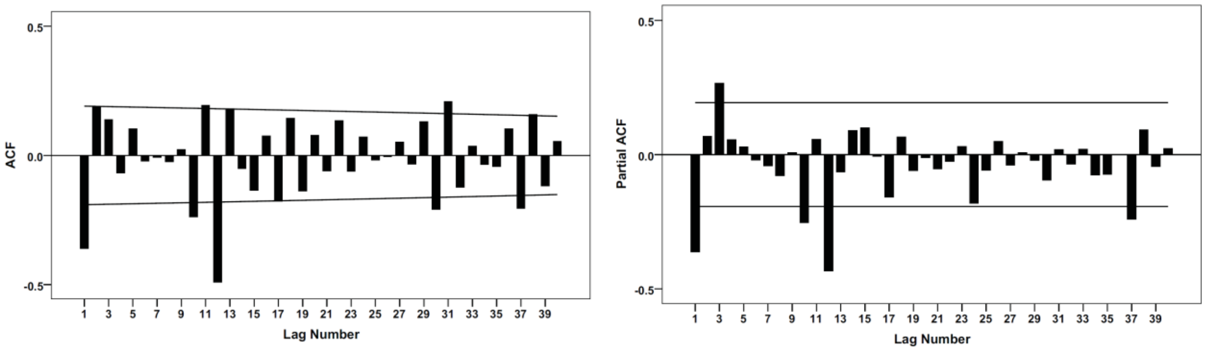

Figure 1: ACF and PACF of IMP at one degree of both non-seasonal and seasonal difference

The summary statistics for the two SARIMA models are shown in Table 2.

Table 2: Summary statistics of 2 SARIMA Models for IMP

	Model Fit statistics				Ljung-Box Q(18)		
Model	R-squared	RMSE	MAPE	MAE	Statistics	DF	Sig.
$SARIMA(1,1,1)(1,1,1)_{12}$	0.728	2876.656	0.0585	2273.735	13.263	14	0.506
$SARIMA(3,1,1)(1,1,1)_{12}$	0.736	2861.977	0.0567	2208.193	8.428	12	0.751

Based on the significance of Ljung-Box Q statistic in Table 2, the two models are all adequate to do the forecasting of IMP. However, $SARIMA(3,1,1)(1,1,1)_{12}$ is finally selected because it has lower MAPE. Figure 2 & Figure 3 illustrate that the residual of $SARIMA(3,1,1)(1,1,1)_{12}$ is really white noise.

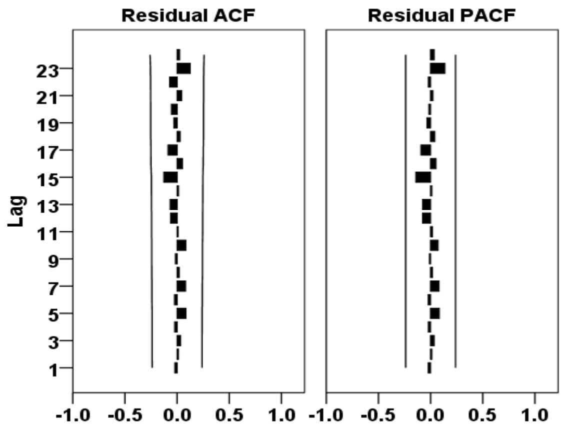

Figure 2: Noise residual ACF and PACF from $SARIMA(3,1,1)(1,1,1)_{12}$ of IMP

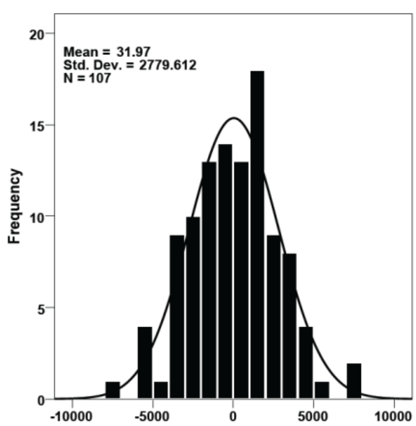

Figure 3: Noise residual from $SARIMA(3,1,1)(1,1,1)_{12}$ of IMP

1.2 Fourier residual modification ARIMA model (ARIMAF)

From the residual series of *SARIMA(3,1,1)(1,1,1)*$_{12}$, a Fourier modified residual series is calculated as per the procedure mentioned in section 2.2. With this modified series, the forecasted values of IMP based on Fourier residual modified *SARIMA(3,1,1)(1,1,1)*$_{12}$ model (*SARIMAF(3,1,1)(1,1,1)*$_{12}$) are obtained based on the equation (7).

Table 3 briefly demonstrates the evaluation indexes of each model with its power in forecasting the volume of imported air cargo in Taiwan.

Table 3: Summary of evaluation indexes of model accuracy

Index / Model	MAPE	S1	S2	C	P	ρ	Forecasting power
SARIMA(3,1,1)(1,1,1)$_{12}$	0.0567	5387.86	2766.59	0.5135	0.8148	0.9433	***Qualified***
SARIMAF(3,1,1)(1,1,1)$_{12}$	0.0104	5387.86	466.76	0.0866	0.9907	0.9896	***Very good***

Based on the indexes in Table 3, it is concluded that Fourier residual modification has improved the forecasting accuracy level of the conventional SARIMA model. Therefore, it is suggested to use *SARIMAF(3,1,1)(1,1,1)*$_{12}$ model for forecasting the volume of imported air cargo in Taiwan. For the monthly volume in year 2012, the forecasted values based on this model are as in column forecasted imports (FIMP) of Table 6.

2. The Volume of Exported Air Cargo in Taiwan

2.1 ARIMA model

The data of the volume of exported air cargo are tested and show that they are not stationary but seasonal. However, at one degree of both non-seasonal and seasonal difference, the series becomes stationary. Figure 4 shows that there are six possible SARIMA models for EXP:

SARIMA(1,1,1)(1,1,1)$_{12}$ *SARIMA(2,1,1)(1,1,1)*$_{12}$ *SARIMA(3,1,1)(1,1,1)*$_{12}$
SARIMA(1,1,3)(1,1,1)$_{12}$ *SARIMA(2,1,3)(1,1,1)*$_{12}$ *SARIMA(3,1,3)(1,1,1)*$_{12}$

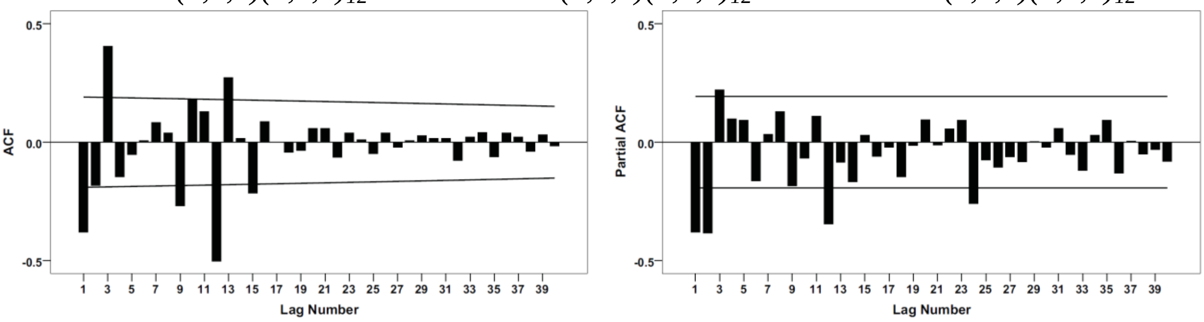

Figure 4: ACF and PACF of EXP at one degree of both non-seasonal and seasonal difference

The summary statistics for the six SARIMA models are shown in Table 4.

Table 4: Summary statistics of 6 SARIMA Models for EXP

Model	Model Fit statistics	Ljung-Box Q(18)

	R-squared	RMSE	MAPE	MAE	Statistics	DF	Sig.
$SARIMA(1,1,1)(1,1,1)_{12}$	0.658	5073.341	0.0662	3488.019	21.031	14	0.101
$SARIMA(2,1,1)(1,1,1)_{12}$	0.690	4850.421	0.0636	3350.076	10.295	13	0.670
$SARIMA(3,1,1)(1,1,1)_{12}$	0.691	4870.054	0.0637	3344.354	10.104	12	0.607
$SARIMA(1,1,3)(1,1,1)_{12}$	0.687	4899.883	0.0635	3340.522	11.123	12	0.518
$SARIMA(2,1,3)(1,1,1)_{12}$	0.692	4885.260	0.0633	3326.270	9.567	11	0.570
$SARIMA(3,1,3)(1,1,1)_{12}$	0.693	4901.227	0.0633	3326.662	9.250	10	0.509

Based on the significance of Ljung-Box Q statistic in Table 4, the six models are all adequate to do the forecasting of EXP. However, $SARIMA(2,1,3)(1,1,1)_{12}$ and $SARIMA(3,1,3)(1,1,1)_{12}$ have the lowest values of MAPE & MAE and Figure 5 to Figure 8 show that their residuals are really white noise; thus, they are finally selected. The residual series from these 2 models are modified with Fourier series in the next stage. After that, one of them will be finally selected based on their MAPE & MAE values.

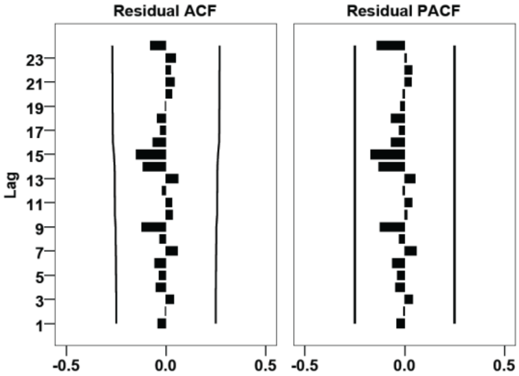

Figure 5: Noise residual ACF and PACF from $SARIMA(2,1,3)(1,1,1)_{12}$ of EXP

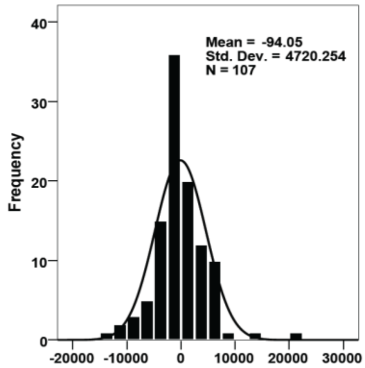

Figure 6: Noise residual from $SARIMA(2,1,3)(1,1,1)_{12}$ of EXP

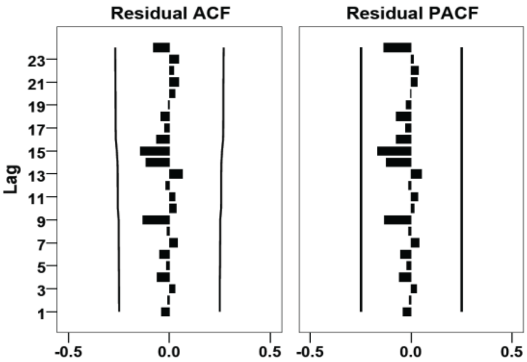

Figure 7: Noise residual ACF and PACF from $SARIMA(3,1,3)(1,1,1)_{12}$ of EXP

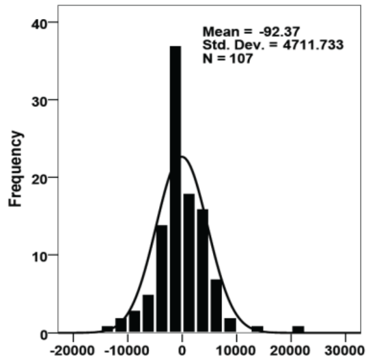

Figure 8: Noise residual from $SARIMA(3,1,3)(1,1,1)_{12}$ of EXP

2.2 Fourier residual modification ARIMA (SARIMA) model (ARIMAF- SARIMAF)

The two residual series of $SARIMA(2,1,3)(1,1,1)_{12}$ and $SARIMA(3,1,3)(1,1,1)_{12}$, are now modified based on the algorithm of Fourier series in section 2.2. These two modified series are then applied to equation (7) to adjust the forecasted values of EXP based on $SARIMAF(2,1,3)(1,1,1)_{12}$ and $SARIMAF(3,1,3)(1,1,1)_{12}$ models, respectively.

Table 5 briefly demonstrates the evaluation indexes of each model with its power in forecasting the volume of exported air cargo in Taiwan.

Table 5: Summary of evaluation indexes of model accuracy

Model \ Index	MAPE	S1	S2	C	P	ρ	Forecasting power
SARIMA(2,1,3)(1,1,1)$_{12}$	0.0633	8464.63	4698.12	0.5550	0.8505	0.9367	***Qualified***
SARIMAF(2,1,3)(1,1,1)$_{12}$	0.0096	8464.63	568.35	0.0671	1.0000	0.9904	***Very good***
SARIMA(3,1,3)(1,1,1)$_{12}$	0.0633	8464.63	4689.67	0.5540	0.8411	0.9367	***Qualified***
SARIMAF(3,1,3)(1,1,1)$_{12}$	0.0096	8464.63	597.56	0.0706	1.0000	0.9904	***Very good***

Table 5 shows that *SARIMAF(2,1,3)(1,1,1)$_{12}$* with the post-error ratio C value of 0.0671 outperforms *SARIMAF(3,1,3)(1,1,1)$_{12}$* and other two SARIMA models in forecasting the volume of exported air cargo in Taiwan. As a result, *SARIMAF(2,1,3)(1,1,1)$_{12}$* is selected. The forecasted values of the volume of exported air cargo in year 2012 based on this model are demonstrated in column forecasted exports (FEXP) in Table 6.

Furthermore, the indexes in Table 5 clearly show that the conventional SARIMA models gain much higher accuracy once their residuals are modified with Fourier series. Along with the case of the volume of imported air cargo discussed in section 4.1.2, it is now affirmably concluded that Fourier residual modification considerably enhances the accuracy level of SARIMA model. Thus, the volume of imported-exported air cargo in Taiwan should be forecasted with SARIMAF models instead of the traditional SARIMA models.

Under SARIMAF models, the forecasted values of the monthly volume of imported and exported air cargo in Taiwan in year 2012 are respectively summarized in column FIMP and FEXP. Consequently, the total monthly traffic of air cargo is illustrated in column FTAC of Table 6.

Table 6: Forecasted volume of air cargo in Taiwan with SARIMAF model (Unit: MT)

Month \ Volume	***Forecasted imports (FIMP)***	***Forecasted exports (FEXP)***	***Forecasted traffic of Air-cargo (FTAC)***
January	37,573	37,268	74,841
February	27,835	27,715	55,550
March	37,435	48,135	85,570
April	42,485	45,834	88,319
May	38,321	43,861	82,182
June	39,073	47,472	86,545
July	46,409	50,504	96,913
August	42,991	48,013	91,004
September	37,762	42,935	80,697
October	40,955	69,157	110,112
November	40,902	34,222	75,124
December	35,708	32,230	67,938

CONCLUSION

ARIMAF/SARIMAF models, which are created by combining the traditional ARIMA/ SARIMA model with the Fourier residual modification, has been proved to be a good model in forecasting the volume of imported-exported air cargo in Taiwan. In this empirical case, it is strongly suggested to use $SARIMAF(3,1,1)(1,1,1)_{12}$ for the import volume and $SARIMAF(2,1,3)(1,1,1)_{12}$ for the export volume due to their low values of MAPE 0.0104 and 0.0096, respectively, which prove that the forecasted values follow closely to the actual figures. These accurate forecasting models will help the authorities and related industries in Taiwan make appropriate plans for the stable development of Taiwan international trade by air.

Fourier residual modification has been successfully applied to the fundamental form of Grey forecasting model GM(1,1) and the ARIMA/ SARIMA models; therefore, it is thought to work well with other forecasting models as well. Further researches on this application should be conducted before the suggestion is firmly verified.

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