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LIFE INSURER FINANCIAL MANAGEMENT: SPACE RISK ANALYSIS IN THE CASE OF POLAND

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ABSTRACT

The aim of this paper is to demonstrate the effect of the regional death risk differentiation approach upon the financial management and profitability of life assurance companies. Such an aim involves answering the question of whether the application of regional death risk differentiation in the process of financial management at insurance companies can improve their financial performance? The above issue was explored on the basis of empirical research undertaken by the author. The research followed the pattern of premium differentiation for the main categories of assurance by regions in Poland. The research reveals that the application of the regional differentiation method indisputably results in the improvement of financial performance of insurance companies, without compromising the fundamental principles of the insurance business, (principle of equilibrium). By employing the regional differentiation method, an insurance company optimises its profitability, while at the same time maintaining the structure of deposits and investments.

INTRODUCTION

The economic processes taking place Ester and Central European countries in recent years has affected also the broad insurance sector. The social and economic transformations, the break up of monopolies and intensifying competition contributed to the development of business insurance. This is demonstrated by both the growing numbers of entities offering insurance products and the increase of the written premium per capita from US$ 5.00 in 1991 to some US$ 300.00 in 2010[1].

Despite such strong growth, the Polish insurance sector is still lagging considerably behind the Western economies. For example Polish insurance market accounts of 1,1% in total European market. The number of life insurance companies operates in Poland is equal to 33 where in UK for example the number reaches the level of 1300[2]. While the Polish insurance market is generally believed to be in the growth stage, it is of particular significance given the integration of Poland with the economic system of the European Union. EU integration and the progressing globalisation of financial services created a need for on-going monitoring and a revision of the current operating strategies, notably in their financial aspect. Many enterprises, mainly in the production sector, do not take out property insurance or refuse to provide life assurance or health insurance cover to their workforce quoting the need to cut costs as the reason.

[1] Data concerning Polish market. Own calculations based on Polish Insurance Chamber Statistics.
[2] *European market in Figures 2011*, CEA Statistics no 44, December 20122

The continued development of the insurance sector is in the interest of the insurance companies operating in Poland. Insurance companies should seek to optimise their financial performance and to improve the flexibility of their insurance services to contribute to improving the financial standing of the insured businesses and households as well. This is possible by an on-going search for factors improving the competitiveness of businesses and increasing insurance awareness among the general public. This is a difficult challenge. One of the success factors is the ability to combine the efforts of the management teams of insurance companies with research findings.

The increasing attractiveness of the insurance market in Poland results in a rapid growth of competition. The tough competition forces insurance companies to take specific actions which should, above all, seek to improve flexibility via product and organisational innovation. The strong competition creates the need to look for ways to optimise the performance of insurers, and to develop on a continuous basis new insurance products that meet the needs of customers. The products should fully meet the needs of customers at the lowest possible insurance premiums.

On the other hand the product should assure financial stability of insurance companies and ultimately ability to pay insurance claims. This is particularly important in the case of life assurance. The long-term nature of life assurance and the special nature of the attendant insurance risk (being the risk of death) renders the financial management at insurance companies offering life assurance products highly difficult and complex. Each decision has specific ramifications for a period of several, or even a dozen or so, following years.

There are few publications, either in Poland or in the world, examining in detail the risk of death and the application of the findings in assurance processes. It is even more difficult to find sources explaining and demonstrating the effect of regional differentiation of insurance (rates) premiums on the financial management of insurance companies. There are a number of research that examine effect on different factors on the purchases of insurance product from costumers side in the case of CEE countries[3]. However it is very difficult to find a study that examines the effect of regional risk differentiation on life insurance companies profitability by affecting insurance rates.

The aim of this paper is to demonstrate the effect of the regional death risk differentiation approach upon the financial management and profitability of life assurance companies. Such an aim involves answering the question of whether the application of regional death risk differentiation in the process of financial management at insurance companies can improve their financial performance?

The above issue was explored on the basis of empirical research undertaken by the author. The research followed the pattern of premium differentiation for the main categories of assurance by regions in Poland. Net premiums were used for comparison, excluding the costs of insurance operations.

[3] Look for example: A.Sliwinski, T.Michalski, M. Rószkiewicz, Demand for Life Insurance—An Empirical Analysis in the Case of Poland, *The Geneva Papers on Risk and Insurance*, 2013, 38, pp. 62-87,

1. Death Risk by regions in Poland

One of the key factors affecting assurance net premium level is the extent of risk covered by insurance, which in this case is the risk of death. The probability of death is a measure of death risk. Based on an examination of changes in the probability of death, one can make certain generalisations and draw consolidated conclusions on the death rate patterns in the population inhabiting a relevant region.

This part of the paper will focus mainly on presenting the differences in the probability of death by region. The differences may appear in the level of the calculated probability of death for individuals of various ages residing in specific regions, and may affect the level of premiums.

The table below presents the death probability figures for men aged from 18 to 30 for the Mazowieckie region and the corresponding figures calculated for the whole of Poland in 1997.

Table 1. Death Probability for Men in Mazowieckie Region vs. Death Probability for Men in Poland.

Age	Mazowieckie	Poland	DIFFERENCE
A	B	C	C-B
18	0.00125	0.00115	**-0.00010**
19	0.00146	0.00131	**-0.00015**
20	0.00124	0.00136	**0.00012**
21	0.00165	0.00136	**-0.00029**
22	0.00175	0.00135	**-0.00040**
23	0.00138	0.00135	**-0.00003**
24	0.00161	0.00138	**-0.00023**
25	0.00163	0.00144	**-0.00019**
26	0.00153	0.00151	**-0.00002**
27	0.00147	0.00159	**0.00012**
28	0.00174	0.00166	**-0.00008**
29	0.00190	0.00174	**-0.00016**
30	0.00210	0.00184	**-0.00026**

Source: Author on the basis of *Trwanie zycia w 1997 (Tabels of life 1997)*, GUS (Main Statistical Office), Warsaw 1998.

As demonstrated clearly by the results, the death probability figures are different for different regions. For example, for a 30-year-old assured individual, the probability of death calculated on the basis of the Dolnoslaskie Region data is 0.001202. The equivalent probability, for example, in the Podlaskie Region is 0.000990, a difference of 17 percent. Similar differences occur for the other regionsand for the regional data-based death probability figures vis-à-vis death probability figures calculated for the whole of Poland, without regional differentiation. The differences are present in both men and women.

The resulting regional differences may cause differentiation of net premiums. One can therefore conclude that by applying the regional differentiation method, insurance companies can shape net premiums as appropriate. The death probability assessment methods and formulae used to calculate premiums are listed in Appendix 1.

2. Regional Comparison of Net Premium – Profitability Regions

The insurance premium embodies the obligation of the insurant towards the insurer for insurance cover during the period of insurance. The insurance premium is therefore the price of the insurance service, and therefore one of the most important considerations taken into account when selecting an insurance company. In emerging markets, where the insurance awareness is low, the premium level often becomes the only selection factor. This approach can bring immeasurable losses if the premium level is without justification established too low. Under the current circumstances, in an attempt to improve their financial performance, insurers should offer lowest practicable premiums ensuring realistic insurance cover to the customers and security to the insurance company.

One of the main objectives of the research presented is to demonstrate the effect of the regional differentiation method upon the level of life assurance premiums. The level of gross premiums is a direct derivative of net premiums. The differences in net premiums calculated on the basis of regional data will result in a proportional reduction of gross premiums. Given the above, the differences in the level of net premiums will likewise affect the final price of the insurance cover service.

In long-term life assurance, the level of net premiums is directly dependent on the average death risk for the period of insurance and the assumed technical interest rate. The net premium is calculated on the basis of the probability of death, assumed longevity and the probable period over which the insurance premium is expected to be paid. The calculations presented are based on the death risk analysis in the individual regions of Poland. The premiums have been calculated on the basis of regional (voivodeship) statistics (NUTS II) and compared with the currently effective net premiums. The Polish insurance market is a relatively young, maturing market. The majority of insurance companies do not maintain in-house statistics and rely on the premium tariffs calculated on the basis of death rate materials pertaining to the national population. The calculations presented apply to "pure" insurance, i.e. whole-life insurance, endowment insurance and pure endowment insurance. The calculations refer to the minimum sums of insurance specified under the general terms of insurance offered by the largest insurance companies operating in Poland. The calculations pertain to individuals from 18 to 35 years of age. This age group is the most numerous group of insurance company customers. The death probability figures assessed on the basis of regional data differ from those established with reference to the national statistics. It is this difference that affects the level of net life assurance premiums.

* R – is the difference between the premium level calculated based on national data and region-specific premiums.

Due to quantitative limitations and in order to maintain a clear structure of the paper, the comparisons below are presented for the Mazowieckie Region. Similar differences are found in the other regions of Poland. The calculation results are shown as charts presenting the relationship of the resulting differences (R)* in the level of premiums relative to the age at which the insurance is taken out. The charts below present the results of calculations and the attendant differences in the case of whole-life insurance, pure endowment insurance and endowment insurance.

Figure 1. Net Premium Comparison – Whole Life Insurance.
Source: Author.

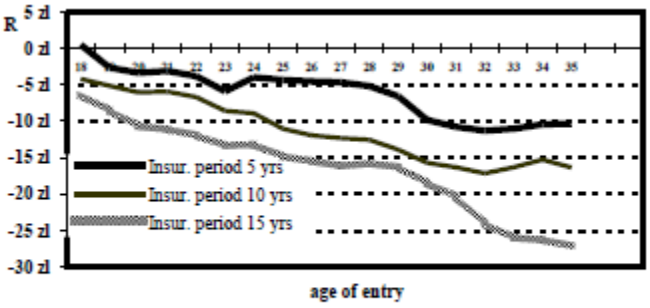


Figure 2. Net Premium Comparison – Pure Endowment Insurance.
Source: Author.

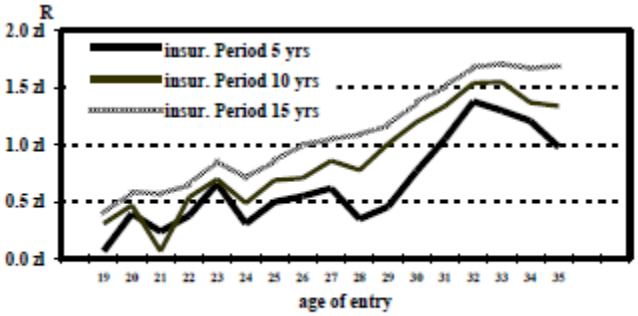


Figure 3. Net Premium Comparison – Endowment Insurance.
Source: Author.

The comparison has shown a difference in the annual net premiums calculated for the national statistics vs. based on the Mazowieckie Region data. The premiums calculated based on the regional data are lower than the premiums based on national statistics. Similar differences apply

to the other regions. While the insurance is taken out for several years and the group of the insured is quite numerous, this may have significant impact on the financial management of insurance companies and their profitability. Employing the regional differentiation method, the life assurance company may, while maintaining the fundamental insurance principles, reduce the level of premiums. The reduction will by no means endanger the financial viability of the insurance company.

In the case of endowment and mixed insurance, premiums calculated on the basis of the voivodeship data differ from the premiums established on the basis of the average national data. In the case of endowment insurance, the level of premiums based on regional data is higher than premiums based on national data. From the point of view of insurance companies, it would be unjustified to apply the regional differentiation method in this type of insurance. In practice, the endowment insurance does not occur as pure insurance. The reason is that, in this case, the insured would lose all the money collected upon his/her death. Mixed-type insurance is more beneficial to the customer.

An analysis of the arising difference reveals a certain pattern. In the case of life assurance, the difference grows with the age of the person taking out the insurance. This regularity does not occur in the case of mixed insurance. In mixed insurance, the resulting differences clearly depend on the period of insurance. For a period of 10 years the difference is twice as big as for five years of the insurance period. The longer the insurance period, the bigger the difference. Here, unlike in the case of life assurance, there is no clear dependence between the premium amount and the age of the insured.

As clearly demonstrated by the research conducted and the comparison between the amount of premiums calculated for regional data vis-à-vis national statistics, the regional differentiation method does affect the level of premiums calculated. As insurance premiums are the key source of income for insurance companies, any significant change in their level is bound to impact significantly upon their financial management.

The findings of the research and the conclusions drawn on the basis of the analysis of regional risk in Poland indicate that Poland has two areas with a similar type of risk. For these areas insurance companies could apply standardised premium tariffs. The list below presents a preliminary division of regional voivodeships together with their characteristics.

their characteristics.

Group I	Characteristics	Group II	Characteristics
Podlaskie		Dolnośląskie	
Podkarpackie		Kujawsko-Pomorskie	
Lubelskie		Warmińsko-Mazurskie	
Łódzkie		Lubuskie	
Świętokrzyskie	Higher average probability of death	Opolskie	Lower average probability of death compared to Group I
Małopolskie		Pomorskie	
Mazowieckie		Wielkopolskie	
		Śląskie	
		Dolnośląskie	
		Zachodniopomorskie	

Source: Author.

For specific ages, the premiums calculated on the basis of Group II regions are lower than the premium based on the average national statistics. The table below presents the irregularity of regional premium distribution as exemplified by life assurance. The irregularity factors were calculated using the following formula:

$$A = \frac{n\sum_{i=1}^{n}(x_i - \bar{x})^3}{(n-1)(n-2)\hat{S}_x^3} \quad (1)$$

where:

n – number of regions,

x_i – premium level in a specific region (voivodeship),

$\bar{x}$ - average premium level,

$\hat{S}$ – standard deviation.

Table 3. Irregularity of Regional Premium Distribution Irregularity as Exemplified by Whole Life Insurance.

Age	Average Premium Level	Skewness
18	0.00451	-0.2195
27	0.00683	0.0089
35	0.01015	0.2057

Based on the analysis presented, an insurance company is capable of optimising the premium level by dividing the country into profitability regions. This requires further detailed statistical research. For the regions defined, the insurer should vary the premium policy, thus maximising the value of the company and its profitability levels.

The calculation of insurance premiums is among the key tasks of the insurance company. This is because premiums, besides benefits and claims, are a principal item of cash flows and the profit and loss account of insurance companies. The premiums paid by insurants are the main item of revenues, while benefits and claims are principal cost items. The operational viability of an insurance company calls for equilibrium between the level of premiums, on the one hand, and the level of benefits and claims, on the other. This is above all due to the fact that the insurance company manages an insurance fund established from the premiums collected. The insurance company's equity is only secondary in balancing its revenues and expenditures. The size of the insurance fund depends on the projected incidence of future events covered by the insurance. One of the basic principles of the insurance company financial management is that of equilibrium between benefits and premiums. This principle requires a balance between the insurance fund, driven by the premium level, and the level of benefits and claims.

FINANCIAL MANAGEMENT AND REGIONAL PREMIUM DIFFERENTIATION

The level of premiums directly affects the size of the insurance fund from which the insurance company meets its obligations in the form of benefits and claims. In the light of the above, as a generalisation, it can be stated that the technical profit generated by the insurance company depends on the size of the fund and the level of benefits and claims paid by it. The increase in the technical profit on the insurance activities will directly affect the overall profit. While the capital structure of the insurance company is maintained, better overall profit will translate into improved profitability. Therefore, if the regional differentiation method increases the value of the balance of the insurance fund – net of benefits and claims – the application of the method will affect the profitability of the insurance company's capital. The impact of the presented method on the value of the insurance fund is shown in Table 4 which compares the course of endowment insurance up to 65 years of age for a group of 10,000 men aged 55, with the insurance amount of PLN 1,000**. The calculation rests on the assumption that the premium paid amounts to PLN 90 and death benefits are payable on 31 December of each year. The technical interest rate is 5 percent. The tables below present results obtained for the Mazowieckie Region. The number of the deceased in a given year follows from the 1999 probability of death calculation for the Mazowieckie Region.

As indicated by the presented data, after the benefits relating to the insurance cover are paid, the application of the regional differentiation method results in an increase of the balance of the insurance fund by some 38.5 percent for the Mazowieckie Region.

** The results of calculations for the Mazowieckie Region were compared with the calculations for such insurance presented in E. Stroiński, *Ubezpieczenia na życie* (*Life Assurance*), LAM, Warsaw 1996, p. 110.

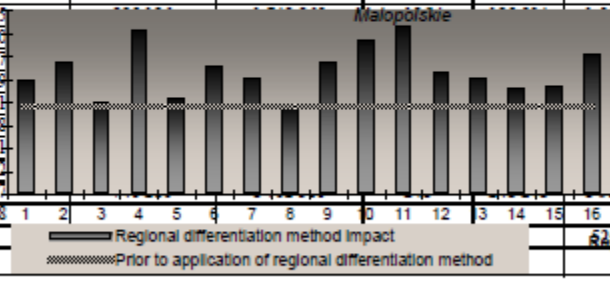
Table 4. **Impact of the Regional Differentiation Method on Insurance Fund Value.**

Age	Number of insured	Current year premium	Current year premium plus residual from previous years after interest	Number of deceased in given year	Value of death benefits	Fund balance net of benefits	Value of endowment benefits
55	10 000	900 000	945 000	173	173 000	772 000	
56	9 827	884 430	1 739 251	183	183 000	1 556 251	
57	9 644	867 960	2 545 422	194	194 000	2 351 422	
58	9 450	850 500	3 362 018	207	207 000	3 155 018	
59	9 243	831 870	4 186 232	220	220 000	3 966 232	
60	9 023	812 070	5 017 217	232	232 000	4 785 217	
61	8 791	791 190	5 855 227	245	245 000	5 610 227	
62	8 546	769 140	6 698 335	257	257 000	6 441 335	
63	8 289	746 010	7 546 712	267	267 000	7 279 712	
64	8 022	721 980	8 401 777	276	276 000	8 125 777	
65	7 746						7 746 000
Balance						**379 777**	

Source: E. Stroiński, *Ubezpieczenia na życie* (*Life Assurance*), LAM, Warsaw 1996, p. 110.

Table 5. **Impact of the Regional Differentiation Method on Insurance Fund Value.**

Age	Number of insured	Current year premium	Current year premium plus residual from previous years after interest	Number of deceased in given year	Value of death benefits	Fund balance net of benefits	Value of endowment benefits
55	10 000	900 000	945 000	165	164 600	780 400	
56						03 271	
57						91 032	
58						29 071	
59						31 470	
60						45 238	
61						07 937	
62						03 933	
63						03 820	
64						4 113	
65	7 988						7 988 261
						146 075	



Source: Author.

Figure 4. Value of Insurance Fund Balance by Region as Exemplified by Endowment Insurance.
Source: Author.

Similar differences are found in the other regions (see Table 6). Figure 4 shows a comparison of the insurance fund balance calculated for the national data and following application of the regional differentiation method. The application of the regional differentiation method results in an increase of the insurance fund balance. A slight decrease was only found in the Lodzkie Region. In the other regions the fund increases on average by 40.7 percent. In the Malopolskie Region the fund was up by 91.41 percent. It should be borne in mind that the analysis rests on a set of assumptions. The assumed number of the insured (10,000) is particularly important. The increases may be even higher for higher numbers of the insured.

The increase of the insurance fund balance has a direct impact upon the technical result of the insurance company. The better the technical financial result, the better the overall performance. Improved financial performance, in terms of both the technical and overall result, affects the profitability of the insurance company. The technical result drives the technical operations profitability ratio, which demonstrates the technical result as a percentage of premium generated on their own contribution. The higher the ratio, the better the standing of the insurance company.

High profitability means a speedy return of the capital employed by the owners to finance the insurance company and a high operational effectiveness. The research findings presented demonstrate that the regional differentiation method, by affecting the insurance company financial management, improves both technical and overall profitability of the business. Assuming that technical operating costs account for 30 percent of total costs, the application of the method would result in a improvement of the return on sales by some 10 percent. Better return on sales directly improves the return on equity and return on total capital of the insurance company. In a free market economy, in a highly competitive environment and where the investment ofcapital by insurance companies is legally restricted, this rate of increase should be seen as significant. The above profitability improvement may strengthen the competitiveness of an insurance company in the market.

Table 6. Value of Insurance Fund Balance by Region.

	Region	Value of insurance fund balance	% difference
1.	Dolnośląskie	PLN 491 045	29.30%
2.	Podlaskie	PLN 572 049	50.63%
3.	Kujawsko-Pomorskie	PLN 394 524	3.88%
4.	Podkarpackie	PLN 708 872	86.65%
5.	Warmińsko-Mazurskie	PLN 412 243	8.55%
6.	Lubelskie	PLN 551 125	45.12%
7.	Lubuskie	PLN 506 850	33.46%
8.	**Łodzkie**	**PLN 376 056**	**-0.98%**
9.	Opolskie	PLN 576 029	51.68%
10.	Świętokrzyskie	PLN 667 569	75.78%
11.	**Małopolskie**	**PLN 726 934**	**91.41%**
12.	Mazowieckie	PLN 525 852	38.46%
13.	Wielkopolskie	PLN 506 686	33.42%
14.	Zachodniopomorskie	PLN 458 554	20.74%
15.	Śląskie	PLN 470 019	23.76%
16.	Pomorskie	PLN 605 923	59.55%

Source: Author.

Summing up, it needs stressing that the application of the regional differentiation method indisputably results in the improvement of financial performance of insurance companies, without compromising the fundamental principles of the insurance business, in particular the principle of equilibrium between benefits and premiums. By employing the regional differentiation method, an insurance company optimises its profitability, while at the same time maintaining the structure of deposits and investments.

The regional differentiation method could be applied in the process of developing a competitive advantage of insurance companies. The application of the method would result in dividing the common European market into the profitability regions, as those referred to above. The division would be based on the analysis of the risk of death in the individual regions of the European Union. The operational strategy driven by profitability regions will help strengthen the competitive edge over other insurers and secure the market position. However, before this strategy can be implemented, extensive research on the death risk in the European Union is necessary. The research should cover a period of several years to eliminate accidental variations. This is a very difficult task requiring financial expenditures as well as collecting a large body of requisite data, as a basis for analysis. The results could help identify European profitability regions for which it would be profitable to apply standardised premium tariffs.

The area of the European Union has been divided into *Nomenclature of Territorial Units for Statistics* (NUTS). The main purpose of the division is to single out territorial units for which statistical data would be collected. The data, collected by Eurostat, is used to assess the economic development in the individual regions. Insurance companies to assess the risk in the NUTS and to calculate insurance premiums could then use the data collected for the individual regions.

At the moment, in the EU there are 77 NUTS I regions, 206 NUTS II regions, 1031 NUTS III regions, 1,074 NUTS IV regions, and 98,433 NUTS V regions. The table below shows the division into NUTS units in the EU countries.

Table 7. **Nomenclature of Territorial Units for Statistics**

Country	NUTS 1		NUTS 2		NUTS 3		NUTS 4		NUTS 5	
Austria	Gruppen von Bundeslaendern	3	Bundeslaender	9	Gruppen von Politischen Bezirken	35	-	-	Gemainden	2351
Belgium	Regions	3	Provinces	11	Arrondissements	43	-	-	Communes	589
Denmark	All country	1	All country	1	Amter	15	-	-	Kommuner	276
Greece	Groups of development regions	4	Development regions	1	Nomoi	51	Eparchies	150	Demoi/Koinotites	5921
Germany	Laender	16	Regierungs-bezirke	38	Kreise	445	-	-	Gemeinden	16176
Spain	Agrupacion de comunidades autonomas	7	Comunidades autonomas + Ceuta i Mellila	18	Provincias + Ceuta i Mellila	51	-	-	Municipios	8077
Finland	Manner-Suomi /Ahvenanmaa	2	Suuralueet	6	Maakunnat	19	Seutukunnat	88	Kunnat	455
France	Z.E.A.T + DOM	9	Regions +DOM	26	Departments + DOM	100	-	-	Communes	36664
Ireland	All country	1	All country	1	Regional Autority Regions	8	Counties/County boroughs	34	DEDs/Wards	3446
Luxemburg	All country	1	All country	1	All country	1	Cantons	12	Communes	118
Holland	Landsdelen	4	Provincies	12	COROP regio's	40	-	-	Gemeenten	672
Portugal	Continente + Regioes autonomas	3	Comissaoes de coordenacao regional + Regioes autonomas	2	Grupos de Concelhos	30	Concelhos-municipios	305	Freguesias	4202
Sweden	All country	1	Riksomraden	8	Lan	24	-	-	Kommuner	286
Italy	Gruppi di regioni	11	Regioni	20	Provincie	103	-	-	Comuni	8100
Great Britain	Standard regions	11	Groups of counties	35	Counties/Local authority regions	65	Districts	485	Wards/Communities/Localities	11095

Source: A.M. Gmyrek, *Nomenklatura statystyczna NUTS-działania dostosowawcze Polski*, „Wspólnoty Europejskie" Nr 6(106) 2000, s.22

The differentiation into the NUTS units could be a factor justifying the regional insurance policy. The research at the NUTS II level also seems justified. While NUTS II and I, have the most extensive statistics, research conducted at the NUTS I level can turn out too general.

By examining the findings of research based on regional data in Poland and given the differentiation into NUTS units in the European Union, one can presume that differentiation of the premiums for the individual regions would have similar financial ramifications for life assurance companies. Using the death rate data in the individual NUTS units, insurance companies could optimise premiums based on an analysis of the insurance risk. This would undoubtedly lead to an increase of the balance of the insurance fund and consequently improve the profitability of the insurance business.

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Data source

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APPENDIX

Appendix 1

I. The probability of death calculated as the incidence of death in a population of people aged "*x*" before they reach the age of "*x+1*". The calculations also take migration into account.

$$q_x = 1-(1-q_x^{'})(1-q_x^{''})$$

In the above formula, the auxiliary variables $q_x^{'}$ and $q_x^{''}$ are calculated as follows:

$$q_x^{'} = \frac{\sum_t D_x^{'}(t)}{\sum[P_x(t-1)-0{,}5*R_{x+1}(t)]}; \qquad q_x^{''} = \frac{\sum_t D_x^{''}(t)}{\sum_t[P_x(t)+D_x^{''}(t)+0{,}5*R_x(t)]}$$

where:

$P_x(t)$ – number of living people aged *x* at the end of year *t*,

$B(t)$ – number of births in year *t*,

$D_x^{'}(t)$ – number of people *x* years of age deceased in year *t* among those born in year *t-x-1*,

$D_x^{''}(t)$ - number of people *x* years of age deceased in year *t* among those born in year *t-x*,

$R_x(t)$ – adjustment of the population due to migration in year *t* of people born in year *t-x*.

The adjustment of the population due to migration was calculated using the following formulae:

$$R_x(t) = [P_{x-1}(t-1) - P_x(t) - D_{x-1}^{'}(t) - D_x^{''}(t)]$$

$$R_0(t) = B(t) - P_0(t) - D_0^{''}(t)$$

II. The premiums were estimated using the following formulae:

1. Whole Life Insurance

The whole-life insurance can be described as the insurer's obligation to pay the party named in the policy upon the insurer's death a pre-agreed sum of money, regardless of whether the death occurs.

In accordance with the principle of equivalence of premiums and benefits:

$$l_x A_x = vd_x + v^2 d_{x+1} + v^3 d_{x+2} + \ldots + v^{w-x} d_w$$

Assumption:
- benefit equal to PLN 1.00.

Therefore, a single premium would be:

$$A_x = \frac{vd_x + v^2 d_{x+1} + v^3 d_{x+2} + \ldots + v^{w-x} d_w}{l_x}$$

Multiplying the numerator and the denominator by v^x gives:

$$A_x = \frac{v^{x+1} d_x + v^{x+2} d_{x+1} + v^{x+3} d_{x+2} + \ldots + v^w d_w}{v^x l_x}.$$

Introducing an additional commutative function C_x instead of $v^{x+1} d_x$, and C_{x+1} instead of $v^{x+2} d_{x+1}$ and applying the commutative function D_x gives:

$$A_x = \frac{C_x + C_{x+1} + C_{x+2} + \ldots + C_{x+w}}{D_x}.$$

In actuarial mathematics a sequence of numbers $C_x+C_{x+1}+C_{x+2}+\ldots+C_{x+w}$ is designated with a symbol (commutative number) M_x. Then the formula is as follows:

$$A_x = \frac{M_x}{D_x}$$

2. Pure Endowment Insurance

Pure endowment insurance is an insurance under which the benefits are paid only to those who have survived the period of insurance.

The equivalence principle can be represented thus:

$$l_x {}_nE_x = v^n l_{x+n}.$$

Solving this formula:

$${}_nE_x = \frac{v^n l_{x+n}}{l_x} = v^n {}_np_x.$$

The formula uses the survival probability for person's aged x in a period of n years. This formula can also be represented as a commutative function in which case the numerator and the denominator of the above formula have to be multiplied by v^x.

Introducing additional commutative functions C_x instead of $v^{x+1} d_x$, and C_{x+1} instead of $v^{x+2} d_{x+1}$ and using the commutative function $D_x = v^x l_x$ gives:

$${}_nE_x = \frac{v^{x+n} l_{x+n}}{v^x l_x} = \frac{D_{x+n}}{D_x}$$

3. Endowment Insurance

A endowment insurance is an insurance under which the benefits are paid upon death in the period of insurance and also when the insured survives the period of insurance.

The endowment insurance can be treated as the sum of periodic insurance and pure endowment insurance. A single premium in the endowment insurance is calculated using the symbol $A_{x:n/}$ and it is the sum of premiums from periodic insurance and pure endowment insurance. This is represented as follows:

$$A_{x:n} = A^1{}_{x:n/} + {}_nE_x.$$

Using the commutative numbers gives:

$$A_{x:n} = \frac{M_x - M_{x+n}}{D_x} + \frac{D_{x+n}}{D_x} = \frac{M_x - M_{x+n} + D_x}{D_x}.$$

Commutative functions:
$C_x = v^{x+1} d_x$,
$C_{x+1} = v^{x+2} d_{x+1}$
$D_x = v^x l_x$

Assumptions:

- single premium,
- benefit equal to PLN 1.00.