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CORRELATED RISK ASSESSMENT AND ITS MANAGERIAL APPLICATIONS

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ABSTRACT

In this paper, we present a new approach of correlated risk assessment by linking the multiple process capability indices and loss functions, in which the multivariate process capability indices and multivariate loss functions describe the likelihood and consequences as a result of nonconformities in multivariate manufacturing or environmental system respectively. Then, the associated relationship equations are developed using multivariate methods. Moreover, a step-by-step procedure is provided to facilitate the implementation of the correlated risk assessment.

Given the multivariate process capability indices, we show that the expected loss can be estimated by our developed relationship equations. Two numerical examples are also given to demonstrate how the correlated manufacturing and environmental risks can be properly assessed by linking the multivariate process capability indices and multivariate loss function. The risk information of likelihood and expected loss classified in the four planning zones of a strategic planning matrix provides practicing managers and engineers with a decision making tool for prioritizing their quality improvement projects when conducting risk assessment for any multivariate process or environmental system. Once the existing quality/environmental problems and their Key Performance Indicators (KPI) are identified, one may conduct risk assessment by applying the relationship equations to evaluate the impact of correlated risk on manufacturing processes or multiple environmental emissions inside company and it can lead to the direction of continuous improvement for any industry.

Keywords: Correlated risk assessment; multivariate process capability indices; multivariate loss functions; quality management; environmental management.

INTRODUCTION

In today's business environment, almost every industrial product has more than one quality characteristic and those characteristics may be correlated. Thus, it is important for companies to evaluate the correlated risks associated with their processes to increase the system safety and product quality. Environmentally speaking, this process of correlated risk analysis can also help reduce hazardous waste being released from their facilities. Traditionally, engineers perform process capability studies to analyze the key characteristic performance by using multivariate process capability indices for measuring process performance. Quality improvement actions can be taken if the process is not capable of meeting specifications. In addition, quality engineers seek to emphasize the urgency of process improvement to senior management by quantifying the likelihood of nonconforming products and their impact on expected quality loss and costs (Pan and Lee, 2010).

Engineering System Risk Assessment (ESRA) systematically evaluates probabilities and consequences of acute, catastrophic failures ((Ayyub, 2003), (KarimiAzari et al., 2011), (Montague, 1990)). According to (Ayyub, 2003), risk is commonly evaluated as the product of the likelihood of occurrence and its impact or severity of its occurrence, namely, RISK= LIKELIHOOD x IMPACT, where likelihood is expressed as the probability of a nonconforming event and the impact represents risk as an expected value or as an average loss. Both (Montague, 1990) and (Ayyub, 2003) have discussed the Quantitative Risk Assessment (QRA) methods at length.

Recently, such quantitative risk assessment methods have been extended to supply chain management (Vilko and Hallikas, 2011) and the “green supply chain” (Wang et al., 2012), which is timely since environmental concerns are drawing much more attention from both academia and industry. Many Statistical Process Control (SPC) tools and principles as applied to quality management are equally useful in achieving environmental improvements. For instance, (Corbett and Van Wassenhove, 1995), and (Madu, 1996, 2004) offer a deep analysis of the broad range of quality control methods that can be applied to environmental management. One benefit of SPC is that it helps operators see and understand problems. Environmental SPC applications also have similar benefits. Operators rarely see the actual, negative impact of pollution caused by a process, but if it can be visualized in real-time through SPC much greater control is possible. (Corbett and Pan, 2002) propose that process capability indices, which measure the degree to which the process is capable of remaining below the existing regulatory limits, can be used as a measure of the environmental quality of a process.

Striking the right balance between tight controls of key processes and still maintaining cost-effectiveness is precisely the purpose of SPC. In order to prevent further environmental contamination, the adoption of SPC environmental risk assessment tools should include process capability indices and an associated loss function for accurately monitoring and evaluating environmental performance. A step towards correlating likelihood with expected loss was taken by (Pan, 2007) as a new loss function-based risk assessment method. It links univariate process capability indices (used to describe the likelihood of nonconforming events) and loss functions (used to describe the impact of such events). The likelihood and consequence resulting from the nonconformance is thus evaluated simultaneously.

The next step then is to build the linkage between the multivariate process capability indices and multivariate loss functions. Hence, this paper explores this relationship especially in terms of the correlated risk of a multivariate manufacturing process as well as assessing the correlated risk for an environmental system. Two numerical examples are given to demonstrate how the correlated manufacturing and environmental risks can be properly assessed by linking the multivariate process capability indices and multivariate loss function.

LITERATURE REVIEW

1. Risk assessment

A well-known classical method for conducting risk assessment is failure mode and effect analysis (FMEA), as described in for instance (Kolarik, 1995). Sometimes a criticality analysis component is added to this, as for instance in the D1-9000 standards at Boeing. This involves identifying each process step that may fail, then assigning rankings for occurrence probability, severity, and detectability. The "occurrence ranking" indicates how likely a failure is considered to be (where higher scores correspond to higher probabilities), and is related to the process capability indices. The "severity ranking" indicates the potential impact of a failure (with higher scores corresponding to more serious impact). The "detectability ranking" indicates how likely it is that a failure can go undetected until its full impact materializes; in the traditional quality control setting, this is the probability of shipping products containing an undetected defect. Higher scores again correspond to higher probability of defects going undetected. The three rankings are then multiplied, and higher total scores indicate higher risk. The other related applications of FMEA can be found in (Wang et al., 2009), (Kenchakkanavar and Joshi, 2010), (Chuang, 2010) and (Nassimbeni et al., 2012).

Risk assessment methods have also ranged from simple classical methods to fuzzy approach mathematical models. (Wang et al., 2009) proposed fuzzy risk priority numbers (FRPNs) for prioritization of failure modes. For solving the risk assessment model selection, (KarimiAzari et al., 2011) used the fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method to solve the risk assessment model selection problem under a fuzzy environment.

2. Multivariate process capability indices

In the past, univariate process capability indices have been used to measure the process performance. Various multivariate statistical methods are now employed when several quality characteristics are interdependent or correlated. (Wang and Chen, 1998) simplified the computation of multivariate process capability by using principal component analysis. (Chan et al., 1991) proposed a multivariate process capability index $\underset{\sim}{C}_{pm}$ using the concept of Mahalanobis distance. (Chen, 1994) proposed a general multivariate capability index that allows elliptical and rectangular specifications. (Foster et al., 2005) later proposed a new multivariate capability index using a process-oriented basis representation.

To develop a quantitative capability measures of a multivariate process in relation to its specifications, (Taam et al., 1993) proposed two multivariate process indices MC_p and MC_{pm}. Their multivariate process capability index MC_{pm} is defined as the ratio of two volumes, i.e.

$$MC_{pm} = \frac{vol.(R_1)}{vol.(R_2)}, \qquad (1.)$$

where R_1 is a modified engineering tolerance region (see Figure 1) and R_2 is a scaled 99.73% process region, which is an elliptical region if the underlying process distribution is assumed to be multivariate normal. Moreover, the modified engineering tolerance region is the largest ellipsoid that is centered at the target and falls within the original engineering tolerance region. Thus, the MC_{pm} index can be rewritten as

$$MC_{pm} = MC_p \frac{1}{D}, \qquad (2.)$$

where $D = (1 + (\boldsymbol{\mu} - \mathbf{T})' \boldsymbol{\Sigma}^{-1} (\boldsymbol{\mu} - \mathbf{T}))^{1/2}$ is a correcting factor if the process mean $\boldsymbol{\mu}$ deviates from the target **T,** $\boldsymbol{\Sigma}$ is process covariance and MC_p is written in Equation (10)

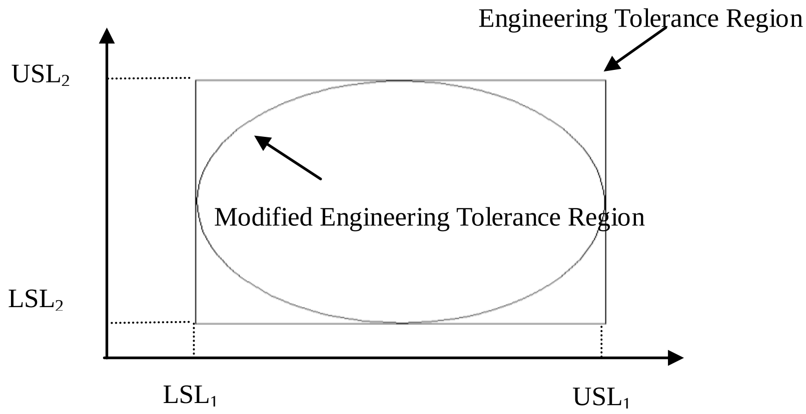

Figure1: Illustration of engineering tolerance region and modified engineering tolerance region.

(Wang and Du, 2000) proposed to use principal component analysis to evaluate the process performance for multivariate data. (Wang et al., 2000) reviewed the three multivariate process capability indices proposed by (Hubele et al., 1991), (Taam et al., 1993) and (Chen, 1994). They pointed out that Hubele's three-component capability vector lacks simplicity and could be confusing in its interpretation and use. Although Taam's MC_{pm} index accurately reflects process variability and centeredness, it does not take into account the correlation between multiple quality characteristics. (Pan and Lee, 2010) revised Taam's modified engineering tolerance region based on the assumption that the correlation of multiple quality characteristics is consistent with the correlation among specifications. The relationship between Taam's modified engineering tolerance region (the regular one) and our revised engineering tolerance region (the slant one) for a process with a bivariate quality characteristic is illustrated in Figure 2.

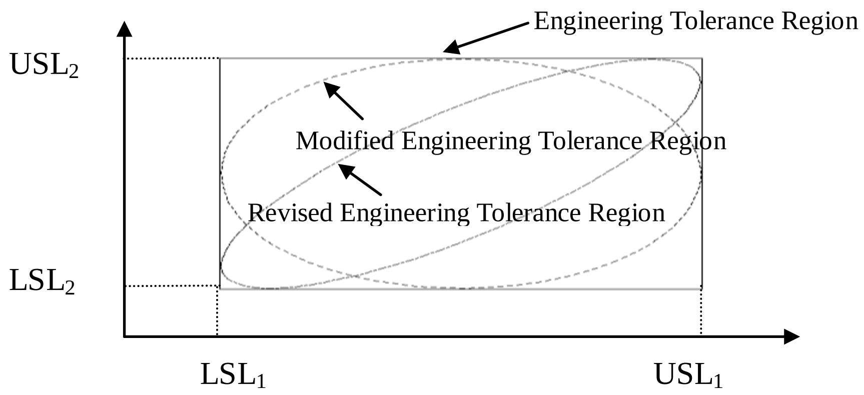

Figure 2. Relationship between Taam's modified region and Pan and Lee's revised engineering tolerance region

To overcome the drawback of overestimation using the MC_p and MC_{pm} indices, (Pan and Lee, 2010) proposed a revised engineering tolerance region (see Figure 2) and redefine new multivariate process capability indices, NMC_p and NMC_{pm} indices. Similar to MC_{pm} ratio of the two volumes shown in Equation (1), the new multivariate process capability index can be defined as

$$NMC_{pm} = \frac{Vol.(E_{d,\mathbf{A}^*,\mathbf{T}})}{Vol.(E_{d,\boldsymbol{\Sigma},\boldsymbol{\mu}})}\left(1+(\boldsymbol{\mu}-\mathbf{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-\mathbf{T})\right)^{-1/2}, \quad (3.)$$

where a revised engineering tolerance region

$$E_{d,\mathbf{A}^*,\mathbf{T}} = \left\{\mathbf{X}\in\mathbf{R}^v \mid (\mathbf{X}-\mathbf{T})'(\mathbf{A}^*)^{-1}(\mathbf{X}-\mathbf{T}) = d^2\right\}, \quad (4.)$$

and where the elements of matrix $\mathbf{A}^*$ are given by

$$\rho_{ij}\left(\frac{USL_i - LSL_i}{2d}\right)\left(\frac{USL_j - LSL_j}{2d}\right) \quad i, j = 1,\ldots,v, \quad (5.)$$

$\mathbf{T}$ is the target vector, ρ_{ij} represents the correlation coefficient between the *i*th and *j*th quality characteristics and $(USL_i - LSL_i)$ denotes the *i*th specification width for each side of rectangle circumscribed to the ellipsoid $E_{d,\mathbf{A}^*,\mathbf{T}}$. The NMC_p index can be used to evaluate the performance of process precision (i.e. the variability in relationship to the revised engineering tolerance region) and the NMC_{pm} index can be used to evaluate both process precision and accuracy (i.e. the deviation from the target).

3. Loss function for multivariate cases

(Pignatiello, 1993) presented a quadratic loss function for multiple-response quality problems. He showed that this function is a generalization of the single-response Taguchi's quadratic loss function. The multivariate loss function can be defined as

$$Q = (\mathbf{X} - \mathbf{T})'\mathbf{C}(\mathbf{X} - \mathbf{T}), \tag{6.}$$

where **X** is a px1 vector of quality characteristics, **C** is a pxp positive definite matrix of costs which represent the losses incurred when **X** deviates from the target vector, **T** is the target vector for p quality characteristics of interest, p is the number of quality characteristics. The quality loss matrix **C** can be expressed as

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1p} \\ C_{21} & C_{22} & \cdots & C_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ C_{1p} & C_{2p} & \cdots & C_{pp} \end{bmatrix}. \tag{7.}$$

Note that if C is a diagonal matrix, then the loss function is the sum of p single-response quadratic loss functions. For non-diagonal **C** matrices, the off-diagonal elements are related to the incremental losses that are incurred when the i th and the j th pair of quality characteristics are simultaneously off-target.

(Pignatiello, 1993) further showed that the expected multivariate loss function can be expressed as:

$$E(Q) = (\boldsymbol{\mu} - \mathbf{T})'\mathbf{C}(\boldsymbol{\mu} - \mathbf{T}) + tr(\mathbf{C}\boldsymbol{\Sigma}). \tag{8.}$$

RELATIONSHIP BETWEEN $MC_{p,pm}$, $NMC_{p,pm}$ CAPABILITY INDICES AND EXPECTED LOSSES

1. Relationship between $MC_{p,pm}$ capability indices and expected loss function

Assuming that the vector **X** of quality characteristics follows a p-dimensional multivariate normal distribution with mean vector $\boldsymbol{\mu}$, variance-covariance matrix $\boldsymbol{\Sigma}$ and its Taam's process capability indices $MC_{p,pm}$ values are given, then the relationship between $MC_{p,pm}$ capability indices and expected loss function can be derived as below:

1.1 The nominal-the- best case

Given the MC_p, MC_{pm} values and $MC_{pm} = MC_p / D$, the relationship equation can be written as

$$\left(\frac{MC_p}{MC_{pm}}\right) = \left(1 + (\boldsymbol{\mu} - \mathbf{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \mathbf{T})\right)^{1/2}, \tag{9.}$$

where the MC_p index represents the ratio of a modified tolerance region with respect to the process variability as written in Equation (10).

$$MC_p = \frac{\left(\prod_{i=q}^{v} r_i\right)\pi^{v/2}\left[\Gamma(v/2)+1\right]^{-1}}{|\boldsymbol{\Sigma}|^{1/2}(\pi K(v))^{v/2}\left[\Gamma(v/2)+1\right]^{-1}}, \tag{10.}$$

where $r_i = (USL_i - LSL_i)/2$, $i = 1,...,v$, USL_i is the upper specification limit for the ith quality characteristic, LSL_i is the lower specification limit for the ith quality characteristic, $|\cdot|$ is a notation of determinant and $\Gamma(\cdot)$ is a Gamma function. Taking the square on Equation (9), one can obtain Equation (11)

$$\left(\frac{MC_p}{MC_{pm}}\right)^2 - 1 = (\boldsymbol{\mu} - \mathbf{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \mathbf{T}). \tag{11.}$$

Equation (9) will vanish if $MC_p = MC_{pm}$, which means the process is on-target and the first term in Equation (8) becomes zero. Thus, we only need to consider the loss due to the process variation and the expected loss function can be reduced to $E(Q) = tr(\mathbf{C}\boldsymbol{\Sigma})$ 。

Referred to the lemma stated in (Johnson and Wichern, 2007), the relationship between determinant and trace can be expressed as Equation (12).

$$tr(\mathbf{B}) = \sum_{i=1}^{p} \frac{|\mathbf{B}|}{|\boldsymbol{\Lambda}_{(i)}|}, \tag{12.}$$

where **B** is a p x p positive definite matrix, $|\boldsymbol{\Lambda}_{(i)}| = \prod_{k=1,k\neq i}^{p} \lambda_k$, λ_i is the i th eigenvalue of **B** matrix. By Equation (12), one can derive the relationship equation between expected quality loss and MC_p index.

$$E(Q) = tr(\mathbf{C}\boldsymbol{\Sigma}) = \frac{1}{(MC_p)^2}\left(|\mathbf{C}||\mathbf{M}|\sum_{j=1}^{p}\frac{1}{|\boldsymbol{\Lambda}_{(j)}|}\right), \tag{13.}$$

where $|\boldsymbol{\Lambda}_{(j)}| = \prod_{k=1,k\neq j}^{p} \lambda_k$, λ_j is the j th eigenvalue of $\mathbf{C}\boldsymbol{\Sigma}$ matrix, $\mathbf{M} = diag\left(r_i^2/\chi^2_{p,0.9973}\right)$, $\chi^2_{p,0.9973}$ is the 99.73% percentile of Chi-square distribution with p degree of freedom.

For the nominal the better case, MC_p and MC_{pm} values will be different if the process mean deviates from the target. To derive the relationship equation between the total expected loss and $MC_{p,pm}$ indices, we consider both the quality loss due to the process variation which can be expressed by Equation (12) and the quality loss due to the process mean deviated from the target which can also been expressed by the $MC_{p,pm}$ values using the following lemma stated in (Johnson and Wichern, 2007).

$$\mathbf{x}'\mathbf{B}\mathbf{x} = \left(\frac{\sum_{i=1}^{p} \lambda_i y_i^2}{\sum_{i=1}^{p} y_i^2} \right) \mathbf{x}'\mathbf{x}, \qquad (14.)$$

where $\mathbf{B}$ is a $p \times p$ positive definite matrix, $\mathbf{x} \neq \mathbf{0}$, λ_i is the i th eigenvalue of $\mathbf{B}$ matrix, $\mathbf{y} = \mathbf{P}'\mathbf{x}$, $\mathbf{P}$ is the orthogonal matrix consisting of p unit eigenvectors.

By Equations (13) and (14), one can derive the following relationship equation between expected loss and Taam's $MC_{p,pm}$ values

$$E(Q) = k_N \left(\left(\frac{MC_p}{MC_{pm}} \right)^2 - 1 \right) + \frac{1}{(MC_p)^2} \left(|\mathbf{C}| |\mathbf{M}| \sum_{j=1}^{p} \frac{1}{|\mathbf{\Lambda}_{(j)}|} \right), \qquad (15.)$$

where k_N is the ratio of " $(\boldsymbol{\mu}-\mathbf{T})'\mathbf{C}(\boldsymbol{\mu}-\mathbf{T})$ divided by $(\boldsymbol{\mu}-\mathbf{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-\mathbf{T})$ ", $|\mathbf{\Lambda}_{(j)}| = \prod_{k=1,k\neq j}^{p} \lambda_k$, λ_j is the j the eigenvalue of $\mathbf{C}\boldsymbol{\Sigma}$ matrix.

Equation (15) implies that given Taam's MC_p , MC_{pm} values and quality loss matrix **C,** the expected quality loss for nominal-the-best case can be estimated.

1.2. The smaller- the- better case

Assuming that the vector $\mathbf{X}$ of quality characteristics follows a p-dimensional multivariate normal distribution with mean vector $\boldsymbol{\mu}$, variance-covariance matrix $\boldsymbol{\Sigma}$, then the MC_{pm} index can be written as Equation (16) for the smaller the better case.

$$MC_{pm} = \frac{MC_p}{\left(1 + \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}\right)^{1/2}}. \qquad (16.)$$

By setting $\mathbf{T}$ =0 in Equation (11), one can obtain Equation (17).

$$\left(\frac{MC_p}{MC_{pm}} \right)^2 - 1 = \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}. \qquad (17.)$$

If the quadratic loss function is considered, then the expected loss can be expressed as Equation (18).

$$E(Q) = \boldsymbol{\mu}'\mathbf{C}\boldsymbol{\mu} + tr(\mathbf{C}\boldsymbol{\Sigma}). \quad (18.)$$

Given $MC_{p,pm}$ values, one can derive the following relationship equation between expected loss and $MC_{p,pm}$ values via Equation (19) for the smaller the better case.

$$E(Q) = k_S\left(\left(\frac{MC_p}{MC_{pm}}\right)^2 - 1\right) + \frac{1}{(MC_p)^2}\left(|\mathbf{C}||\mathbf{M}|\sum_{j=1}^{p}\frac{1}{|\boldsymbol{\Lambda}_{(j)}|}\right), \quad (19.)$$

Where k_S is the ratio of $\boldsymbol{\mu}'\mathbf{C}\boldsymbol{\mu}$ divided by $\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$, $|\boldsymbol{\Lambda}_{(j)}| = \prod_{k=1,k\neq j}^{p}\lambda_k$, λ_j is the j the eigenvalue of $\mathbf{C}\boldsymbol{\Sigma}$ matrix。

Equation (19) implies that given Taam's MC_p , MC_{pm} values and quality loss matrix **C,** the expected quality loss for smaller- the- better case can be estimated.

2. Relationship between $NMC_{p,pm}$ capability indices and expected loss function

Similar to the proof as stated in section 3.1, one can derive the following relationship equation between expected loss and $NMC_{p,pm}$ values hold for the nominal-the-better case:

$$E(Q) = k_N\left(\left(\frac{NMC_p}{NMC_{pm}}\right)^2 - 1\right) + \frac{1}{(NMC_p)^2}\left(|\mathbf{C}||\mathbf{A}^*|\sum_{j=1}^{p}\frac{1}{|\boldsymbol{\Lambda}_{(j)}|}\right), \quad (20.)$$

where k_N is the ratio of " $(\boldsymbol{\mu}-\mathbf{T})'\mathbf{C}(\boldsymbol{\mu}-\mathbf{T})$ divided by $(\boldsymbol{\mu}-\mathbf{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-\mathbf{T})$ ", $|\boldsymbol{\Lambda}_{(j)}| = \prod_{k=1,k\neq j}^{p}\lambda_k$, λ_j is the j the eigenvalue of $\mathbf{C}\boldsymbol{\Sigma}$ matrix. In Equation (4), if we let $d^2 = \chi^2_{1-\alpha,v}$, then the NMC_p index can be written as

$$NMC_p = \frac{|\mathbf{A}^*|^{1/2}\left(\pi\chi^2_{1-\alpha,v}\right)^{v/2}\left[\Gamma\left(\frac{v}{2}+1\right)\right]^{-1}}{|\boldsymbol{\Sigma}|^{1/2}\left(\pi\chi^2_{1-\alpha,v}\right)^{v/2}\left[\Gamma\left(\frac{v}{2}+1\right)\right]^{-1}} = \left(\frac{|\mathbf{A}^*|}{|\boldsymbol{\Sigma}|}\right)^{1/2} \quad (21.)$$

and the NMC_{pm} index can be written as

$$NMC_{pm} = NMC_p / D, \quad (22.)$$

where $D = (1 + (\boldsymbol{\mu} - \mathbf{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu} - \mathbf{T}))^{1/2}$. The term D in Equation (22) denotes a function of Mahalanobis distance between the process mean and target vector $\mathbf{T}$. It can be used to measure the process deviation from target vector $\mathbf{T}$. Note that the univariate C_p and C_{pm} indices can be considered as a special case of new multivariate NMC_p and NMC_{pm} indices if $v = 1$ and $1 - \alpha = 0.9973$.

Equation (20) implies that given Pan and Lee's NMC_p, NMC_{pm} values and quality loss matrix **C,** the expected quality loss for nominal-the-best case can be estimated.

If the quadratic loss function is considered, then the expected loss can be expressed as Equation (18). Given NMC_p and NMC_{pm} values, one can derive the following relationship equation between expected loss and $MC_{p,pm}$ values by Equation (23) for the smaller the better case.

$$E(Q) = k_S\left(\left(\frac{NMC_p}{NMC_{pm}}\right)^2 - 1\right) + \frac{1}{(NMC_p)^2}\left(|\mathbf{C}|\left|\mathbf{A}^*\right|\sum_{j=1}^{p}\frac{1}{\left|\boldsymbol{\Lambda}_{(j)}\right|}\right), \qquad (23.)$$

Where k_S is the ratio of $\boldsymbol{\mu}'\mathbf{C}\boldsymbol{\mu}$ divided by $\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$, $|\boldsymbol{\Lambda}_{(j)}| = \prod_{k=1,k\neq j}^{p}\lambda_k$, λ_j is the j the eigenvalue of $\mathbf{C}\boldsymbol{\Sigma}$ matrix. Equation (23) implies that given Pan and Lee's NMC_p, NMC_{pm} values and quality loss matrix **C,** the expected quality loss for smaller-the-better case can be estimated.

To facilitate practitioners in conducting the correlated risk assessment, a step-by-step procedure is listed as below.

1. Perform a multivariate process capability study with the required sample size.
2. Collect measurement data and perform a normality test for the collected data.
3. Calculate the new NMC_p and NMC_{pm} indices using Equations(21) and (22).
4. Given the new NMC_p and NMC_{pm} indices, calculate the expected losses for either nominal-the-best or smaller-the-better cases using Equations (20) and (23) respectively.
5. Perform the sensitivity analysis of risk (obtain various expected losses under different hypothetical process capability indices) and then setup a realistic goal for future quality improvement.
6. Evaluate the risk information of likelihood (reflected by process capability indices) and expected losses (represent impact) for different multivariate processes. Then prioritize various quality improvement projects by using the strategic planning matrix as shown in Figure 5.

NUMMERICAL EXAMPLES

Example 1 Manufacturing risk assessment using Sultan's manufacturing data.

(Sultan, 1986) discussed an example in which the Brinell hardness (H) and tensile strength (S) are two quality characteristics for an industrial product. The engineering tolerances for H and S are given by (112.7, 241.3) and (32.7, 73.3) respectively and the target vector of H and S is $\mathbf{T}' = [177, \quad 53]$. After collecting 25 measurements as listed in Table 1, a multivariate process capability study is conducted (assuming the process is in control).

Table 1. The 25 measurements of Brinell hardness (H) and tensile strength (S) for an industrial product.

H	S	H	S	H	S
143	34.2	141	47.3	178	50.9
200	57.0	175	57.3	196	57.9
160	47.5	187	58.5	160	45.5
181	53.4	187	58.2	183	53.9
148	47.8	186	57.0	179	51.2
178	51.5	172	49.4	194	57.5
162	45.9	182	57.2	181	55.6
215	59.1	177	50.6		
161	48.4	204	55.1		

By performing Shapiro-Wilk test, we found that the 25 collected measurements follow a multivariate normal distribution with the sample mean vector $\bar{\mathbf{X}}' = [177.2, \quad 52.32]$ and the sample covariance matrix $\mathbf{S}$, where

$$\mathbf{S} = \begin{bmatrix} 338 & 88.8925 \\ 88.8925 & 33.6247 \end{bmatrix}.$$

Then, the matrix $\mathbf{A}^*$ can be obtained as below:

$$\mathbf{A}^* = \begin{bmatrix} 349.52131 & 92.01022 \\ 92.01022 & 34.83724 \end{bmatrix} \qquad \text{(Based on Equation (5))}$$

The actual relationship among the 99.73% process region, 99.73% revised engineering tolerance region and engineering tolerance region is illustrated in Figure 3.

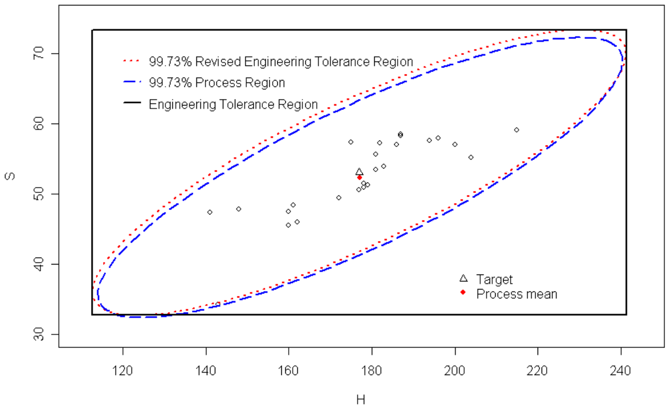

Figure 3. Relationship among 99.73% revised engineering tolerance region, 99.73% process region and engineering tolerance region in example 1.

Apparently, the process mean is close to the target and the "99.73% process region" is approximately equal to the "99.73% revised engineering tolerance region". The comparison results of using various multivariate process indices for estimating the performance of an industrial product are summarized in Table 2. Since the estimated conforming rate for this example is 99.91% under the assumption of multivariate normality for the underlying process distribution and the new indices $NMC_p = 1.0351$ and $NMC_{pm} = 1.0087$, based on Equations (21) and (22) respectively, are nearly equal to 1; which indicates that the 99.73% process region is close to the 99.73% revised engineering tolerance region and the process mean is close to the target (see Figure 3). Thus, the true process performance can be correctly reflected by the new indices NMC_p and NMC_{pm}, i.e. the process is capable. Whereas, the process capability is overestimated by Taam's two new capability indices $MC_p = 1.8751$ and $MC_{pm} = 1.8272$, based on Equations (10) and (2) respectively, since the correlation among multiple quality characteristics are not taken into account. Thus, it is suggested that the new indices NMC_p and NMC_{pm} be used in assessing the likelihood of nonconforming. Furthermore, assuming that the quality loss is \$0.8 per unit when the hardness value is deviated from the target and the quality loss is \$1 per unit when the strength value is deviated from the target, while the quality loss is \$0.89 per unit when both characteristics are deviated from the target, the quality loss matrix can be expressed as：

$$\mathbf{C} = \begin{bmatrix} 0.8 & 0.89 \\ 0.89 & 1 \end{bmatrix}, \qquad \text{(Based on Equation (7))}$$

, $|\mathbf{A}^*| = 3710.48$, $|\mathbf{C}| = 0.0079$ and $k_N = 4.8303$ (Based on the ratio of $(\boldsymbol{\mu}-\mathbf{T})'\mathbf{C}(\boldsymbol{\mu}-\mathbf{T})$ divided by $(\boldsymbol{\mu}-\mathbf{T})'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}-\mathbf{T})$).

Given the multivariate process capability indices $NMC_p = 1.0351$ and $NMC_{pm} = 1.0087$ (as listed in Table 2), the expected quality loss can be estimated by Equation (20), i.e.

$$E(Q) = k_N\left(\left(\frac{NMC_p}{NMC_{pm}}\right)^2 - 1\right) + \frac{1}{\left(NMC_p\right)^2}\left(|\mathbf{C}||\mathbf{A}^*|\sum_{j=1}^{2}\frac{1}{|\mathbf{\Lambda}_{(j)}|}\right)$$
$$= 4.8303\left(\left(\frac{1.0351}{1.0087}\right)^2 - 1\right) + \frac{1}{(1.0351)^2}\left(0.0079 \times 3710.48\left(\frac{1}{0.0592} + \frac{1}{462.1942}\right)\right),$$
$$= 0.256 + 462 = 462.256$$

which is the same as the expected loss calculated by Equation (8), i.e.

$$E(Q) = (\boldsymbol{\mu} - \mathbf{T})^T \mathbf{C}(\boldsymbol{\mu} - \mathbf{T}) + tr(\mathbf{C}\boldsymbol{\Sigma}) = 0.256 + 462 = 462.256\,.$$

Hence, we demonstrate that the calculation of expected quality loss using the relationship equation between multivariate process capability indices and loss functions is accurate. One advantage of using our proposed relationship equation is that the impact of process capability indices on the $E(Q)$ can be clearly understood by quality practitioners. The other advantage is that the risk assessment conducted in this way can facilitate the sensitivity analysis of risk and then setup a realistic goal for future quality improvement. For example, if the improvement measures successfully applied to the existing process (given that its NMC_p and NMC_{pm} indices have been increased to 1.5), then the expected quality loss can be reduced to \$ 220, i.e.

$$E(Q) = k_N\left(\left(\frac{NMC_p}{NMC_{pm}}\right)^2 - 1\right) + \frac{1}{\left(NMC_p\right)^2}\left(|\mathbf{C}||\mathbf{A}^*|\sum_{j=1}^{2}\frac{1}{|\mathbf{\Lambda}_{(j)}|}\right)$$
$$= 4.8303\left(\left(\frac{1.5}{1.5}\right)^2 - 1\right) + \frac{1}{(1.5)^2}\left(0.0079 \times 3710.48\left(\frac{1}{0.0592} + \frac{1}{462.1942}\right)\right) = \$220$$

On the other hand, if the management team wants to reduce the quality loss in half, then the team needs to setup a goal for the quality improvement project to bring NMC_p and NMC_{pm} index values up to 1.5. If the continued quality improvement measures have been successfully applied to the existing process (given that both NMC_p and NMC_{pm} indices have been increased to 2.0), then the expected loss will be lowered to only \$116, i.e.

$$E(Q) = k_N\left(\left(\frac{NMC_p}{NMC_{pm}}\right)^2 - 1\right) + \frac{1}{\left(NMC_p\right)^2}\left(|\mathbf{C}||\mathbf{A}^*|\sum_{j=1}^{2}\frac{1}{|\mathbf{\Lambda}_{(j)}|}\right)$$
$$= 4.8303\left(\left(\frac{2}{2}\right)^2 - 1\right) + \frac{1}{(2)^2}\left(0.0079 \times 3710.48\left(\frac{1}{0.0592} + \frac{1}{462.1942}\right)\right) = \$116$$

Table 2. The basic statistics and multivariate process capability indices in example 1.

Basic statistics for H and S[a]	Process capability indices
Sample Mean for H=177.2	MC_p =1.8751
Sample Mean for S=52.32	MC_{pm} =1.8272
Standard deviation for H=18.385	NMC_p =1.0351
Standard deviation for S=5.799	NMC_{pm} =1.0087
Correlation coefficient between H and S=0.834	

[a] H represents hardness, S represents tensile strength

This numerical example shows that by linking the multivariate process capability indices and multivariate loss function, one can perform the sensitivity analysis between the expected loss and process capability indices. Since the expected loss represents impact and process capability indices reflect the likelihood/opportunity of product failures, the simultaneous consideration of impact and likelihood provides a useful guideline for quality practitioners in performing the correlated risk assessment for any multivariate manufacturing process.

Example 2 Environmental risk assessment using the air pollution data from Song-Shan station.

There are 58 surveillance stations established by EPA of Taiwan to monitor the air quality in Taiwan. Song-Shan is an administrative district in Taipei city; the capital of Taiwan and a domestic airport is located there. It is also well known for a long history of higher pollution in Northern Taiwan region. The air quality data of PM10 and O3 collected by Song-Shan station are discussed since PM10 and O3 have potential threat to health. The recent hourly data were collected by Song-Shan station. In this example, daily average is used and we drop some missing data due to measurement failure. The basic statistics of air pollution data at Song-Shan station are summarized in Table 3. We found that the original air pollution data for PM10 and O3 did not follow a bivariate normal distribution at α=0.05. After performing the Box-Cox normal transformation method proposed by (Andrews et al., 1971), the scatter plot of air pollution data at Song-Shan station shows that the transformed data follows a bivariate normal distribution (see Figure 4). Note that we take an exponent of 0.6248 to the original O3 data and an exponent of 0.1933 to the original PM10 data for normal transformation.

Table 3. The basic statistics of air pollution data at Song-Shan station.

Air Pollutants	Sample means	Std. deviation	Correlation coeff.
O3	24.5204	10.2247	0.2694
PM10	58.6192	26.7534	

The upper specification limit (USL) for O3 stipulated by the EPA of Taiwan is set to the hourly average = 0.12 p.p.m., while the USL for PM10 is set to the daily average = 125 $\mu g/m^3$. After calculating the $MC_{p,pm}$ and $NMC_{p,pm}$ index values for the transformed data, the comparison results of using various multivariate process indices for estimating the O3 and PM10 air pollutants at Song-Shan district are summarized in Table 4.

Table 4. The basic statistics and multivariate process capability indices of O3 and PM10 after normal transformation.

Basic statistics for air pollutants	Process capability indices
Sample mean for O3=7.2240	MC_p =11.6806
Sample mean for PM10 =2.1613	MC_{pm} =1.0392
Standard deviation for O3=1.9514	NMC_p =11.3181
Standard deviation for PM10=0.1938	NMC_{pm} =1.0070
Correlation coefficient between O3 and PM10=0.2472	

Assume that the additional cost per patient due to the increase of respiratory diseases (caused by PM10 and O3 in Song-Shan district) = \$83.36 when only O3 exceeds the USL; an additional cost per patient due to the increase of respiratory diseases =\$53.16 when only PM10 exceeds the USL; and an additional cost per patient due to the increase of respiratory diseases = \$66.45 when both PM10 and O3 exceed the USL, then the environmental loss matrix can be expressed as:

$$\mathbf{C} = \begin{bmatrix} 83.36 & 66.45 \\ 66.45 & 53.16 \end{bmatrix}, \qquad \text{(Based on Equation (9))}$$

and $|\mathbf{A}^*| = 17.2000$ (Based on Equation (5)) , $|\mathbf{C}| = 15.8151$, $k_s = 53.2463$ (the ratio of $\boldsymbol{\mu}'\mathbf{C}\boldsymbol{\mu}$ divided by $\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$).

Based on Equations (21) and (22), NMC_p = 11.3181, NMC_{pm} = 1.0070 (as listed in Table 4), then the expected daily environmental loss can be estimated by Equation (23), i.e.

$$E(Q) = k_s\left(\left(\frac{NMC_p}{NMC_{pm}}\right)^2 - 1\right) + \frac{1}{(NMC_p)^2}\left(|\mathbf{C}||\mathbf{A}^*|\sum_{j=1}^{2}\frac{1}{|\boldsymbol{\Lambda}_{(j)}|}\right)$$
$$= 53.2463\left(\left(\frac{11.3181}{1.0070}\right)^2 - 1\right) + \frac{1}{(11.3181)^2}\left(15.8151 \times 17.2000\left(\frac{1}{0.0064} + \frac{1}{331.8355}\right)\right)$$
$$= 6673 + 332 = 7005$$

In other words, the average daily cost increase of medical expenses in Song-Shan district in terms of the likelihood of respiratory diseases/multivariate process capability indices = \$7005, which is the same as the expected loss calculated by Equation (8), i.e.

$$E(Q) = (\boldsymbol{\mu} - \mathbf{T})^T \mathbf{C}(\boldsymbol{\mu} - \mathbf{T}) + tr(\mathbf{C}\boldsymbol{\Sigma}) = 6673 + 332 = 7005$$

Note that MC_p = 11.6806, MC_{pm} = 1.0392 (based on Equations (10) and (2) respectively) are very close to NMC_p = 11.3181, NMC_{pm} = 1.0070 due to the low correlation coefficient (=0.269) between O3 and PM10. Moreover, significant gaps exist in between MC_p = 11.6806 and MC_{pm} = 1.0392 as well as in between NMC_p = 11.3181 and NMC_{pm} = 1.0070, which indicate the existing levels for the air pollutants of PM10 and O3 are much higher than the target level of zero though only a few points exceeding the USL as shown in Figure 4. This numerical example further shows that by linking the multivariate process capability indices and multivariate loss function, the simultaneous consideration of expected loss (represents impact) and process capability indices (reflect likelihood of failure) provides practitioners a useful guideline in performing the correlated risk assessment for any environmental system.

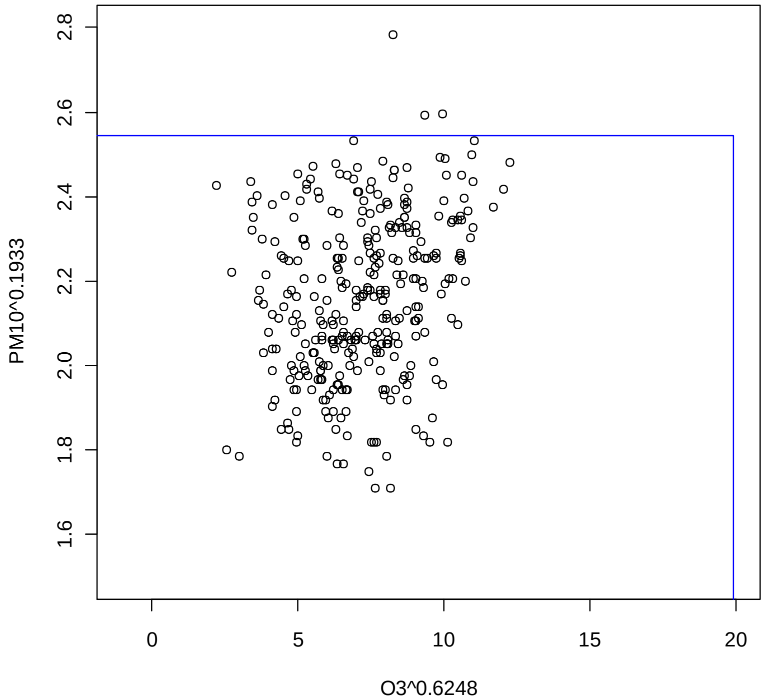

Figure 4. The scatter plot of air pollution data with upper specification limits for O3 and PM10 at Song-Shan station (after normal transformation)

STRATEGIC INSIGHTS AND MANAGERIAL IMPLICATIONS

After obtaining the relevant risk information of likelihood and expected losses from the key multivariate processes, practicing managers and engineers can categorize quality or environmental improvement projects in the following four planning zones of a strategic planning matrix.

(a) Priority zone – high impact (in terms of high expected loss) and high likelihood (in terms of low capability indices).
(b) Long-term zone – high impact and low likelihood (in terms of high capability indices).
(c) Contingency zone – low impact (in terms of low expected loss) and high likelihood.
(d) Non-priority zone – low impact and low likelihood

The strategic planning matrix containing four zones as shown in Figure 5 is used for these assessments.

		Impact & Importance: Low	Impact & Importance: High
Likelihood	High	**Contingency Zone**	**Priority Zone**
	Low	**Non-Priority Zone**	**Long-Term Zone**

Figure 5. The classification of four planning zones based on the impact and likelihood

It is worthy to note that (1) special attention and immediate corrective action efforts need to be taken if quality improvement projects fall into Priority Zone (2) long-term planning and proactive action efforts shall be taken if quality improvement projects fall into Long-Term Zone (3) contingency plan need to be formulated to cope with the emergent conditions if quality improvement projects fall into Contingency Zone (4) maintain the status quo, i.e. the practitioners do not need to pay too much attention if quality improvement projects fall into Non-Priority Zone.

The risk information of likelihood and expected loss classified into four quadrants helps practitioners to prioritize quality improvement projects when conducting correlated risk assessment for any multivariate process or environmental system. Once the quality improvement projects are prioritized, the new approach of correlated risk assessment can lead to the direction of continuous improvement for any industry.

In terms of industry use, it is unlikely that smaller corporations that have difficulty applying FMEA to their operations could benefit from correlated risk analysis. A commitment to SPC and its processes is the most conducive environment for this advanced process monitoring. As well, industries whose products involve complicated components and assemblies such as aerospace, automotive or military contracting have the most to gain from correlated risk analysis. On the environmental side, chemical processing industries such as petro refineries, steel plants or plastics manufacturing could benefit by verifying that their output of pollutants is well within governmental tolerance and regulation.

CONCLUSIONS

Correlated risk assessment with its emphasis on the loss function is an essential tool for multi-response quality engineering. In this paper, we propose a new approach of correlated risk assessment by linking the multiple process capability indices and loss functions. With the multivariate process capability indices, we have shown that the expected quality loss can be estimated for both the-nominal-the-best and the-smaller-the-better cases. The two numerical examples demonstrate that expected losses can be estimated by various multivariate process capability indices and they show the advantage of using our proposed relationship equations.

The new multivariate process capability indices properly reflect the actual nonconforming rate.

To implement correlated risk assessment in practical applications, existing quality or environmental issues must be identified. Then, Key Performance Indicators (KPIs) and their multivariate process capability indices are developed to quantify the system. At this point risk can be assessed by applying the new relationship equations to evaluate the impact of correlated risk on manufacturing processes, or on multiple environmental emissions, such as SO_2 and NO.

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